COMMON CORE STATE STANDARDS

CURRICULUM FRAMEWORK PROGRESS GUIDE

Elementary Mathematics

Grade 3

Prince George's County Public Schools

PGIN 7690-0949

Board of Education of Prince George's County, Maryland

BOARD OF EDUCATION
OF
PRINCE GEORGE'S COUNTY, MARYLAND

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Rationale

The purpose of the Prince George's County Common Core State Standards Curriculum Framework Progress Guide for Elementary Mathematics, (CCSS/CFPG) is to provide grade level teachers with a means of sequencing grade level mathematics instruction that is based upon the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are designed to provide a focused and coherent set of content and practice standards that require in depth instruction that leads to deeper student understanding. Throughout the standards, the use of the term "understand" is incorporated to ensure students are learning and absorbing the critical information needed to succeed as they proceed to higher levels of mathematics education. The expectations at each grade level include student mastery of critical mathematical concepts, procedural skills and attainment of grade level fluencies.

The CCSS/CFPG establishes Units for each quarter with guiding timeframes and a set of resources to support delivery of instruction. It is the expectation for teachers to develop lessons and activities that are designed and modified for their specific student population. Teachers may design formative and summative assessments that measure student mastery of concepts and skills. Mastery of grade level standards is expected by the end of the grade level. Individual standards cannot be thought of as a checklist and will be taught multiple times within the year. Therefore, standards that have been addressed in a summative assessment may not necessarily be listed in subsequent Units, even though the standard may be embedded within your lesson.

Terms Used in the Curriculum Framework Progress Guide

- **PARCC:** The Partnership for Assessment of Readiness for College and Careers is a group of states working together to create common assessments in English and mathematics for grades K-12. The PARCC assessments will be ready for states to use in the 2014-15 school year.
- **PARCC Content Cluster:** Describes content emphasized in the standards at the cluster level for each grade. To make the standards more transparent and useful, they are designated as Major, Additional and Supporting. Some clusters that are not major emphases in themselves are designed to support and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade are called additional.
- **Domain:** Larger groups of related standards. Standards from different domains may sometimes be closely related.
- **Cluster:** Groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
- **Standard:** Defines what students should understand and be able to do.
- **Fluency:** Expectation at each grade level for students to perform given mathematics problems and or facts "fast and accurately". For first grade, students should be fluent with addition and subtraction within10. (Known facts to 10) A consideration for advanced learners is that students must be able to mentally add and subtract within 20, and on paper within 100, by the end of second grade.
- **Coherence:** The steps of learning, which are sequential and developmentally appropriate, to understand the content and processes of mathematics. Within the Common Core State Standards, the 8 mathematical practices remain the same throughout the grades, while the content builds like steps in a staircase.
- **Focus:** A set of lessons, focused on or more domains, which spans the time of a quarter of the school year.
- **Content Tracker:** Tracking tool to monitor student progress based on specific skills taught quarterly.
- **Unit:** A selection of Common Core standards, which should be taught over a given period of time within a quarter.
- **Conceptual Understanding:** Consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles.
- **Essential Questions:** A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.
- **Enduring Understandings:** Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

Codes for Common Core State Standards: K – 12

Abbreviation	Term
G	Geometry
MD	Measurement & Data
NBT	Number & Operations in Base Ten
OA	Operations and Algebraic Thinking
NF	Number and Operation - Fractions

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them

6. Attend to precision

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics
5. Use appropriate tools strategically

7. Look for and make use of structure
8. Look for and express regularity in

Overarching habits of mind of a productive mathematical thinker

Reasoning and Explaining

Modeling and Using Tools

Seeing Structure and Generalizing

Grade 3 Common Core State Standards for Mathematical Practice

Student Friendly Version of Standards for Mathematical Practices (SMP)

Standards	Explanations and Examples	Possible Questions and Prompts:	Student "Look Fors"
1. Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualized and solve problems. They may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.	• How would you describe the problem in your own words? • What facts do you have? What do you know that is not stated in the problem? • How did you tackle similar problems? • Would it help to create a diagram? …make a table? …draw a picture? • What strategies are you going to use?	❑ Consider or attempt multiple entry points to its solution ❑ Analyze information (givens, constraints, relationships, goals) ❑ Make conjectures and plan a solution pathway ❑ Use objects, drawings, and diagrams to solve problems ❑ Monitor progress and change course as necessary ❑ Check answers to problems and ask, "Does this make sense?"
2. Reason abstractly and quantitatively.	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.	• Can you tell why that is true? • How did you reach your conclusion? • How does your answer connect to the question? Does it make sense? • Can you make a model to show that?	❑ Make sense of quantities and relationships in problem situations ❑ Represent abstract situations symbolically ❑ Create a coherent representation of the problem ❑ Translate from contextualized to generalized or vice versa ❑ Flexibly use properties of operations
3. Construct viable arguments and critique the reasoning of others.	In third grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.	• Can you convince the rest of us that your answer makes sense? • What do you think about what _____ said? • Do you agree? Why or why not? • Does anyone have the same answer but a different way to get it? • Can you explain what____ is saying? Can you explain why his/her strategy works? • What don't you understand about what ____ is saying?	❑ Use definitions and previously established causes/effects (results) in constructing arguments ❑ Make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas ❑ Listen to or read the arguments of others ❑ Ask probing questions to other students
4. Model with mathematics.	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.	• How would you model the situation with a diagram, picture, table, graph, equation, or words? • Can you use color, words, or diagrams to show the connections between these ideas? • How do the different models connect or relate to each other (i.e. table to graph, graph to equation)?	❑ Determine an equation that represents a situation ❑ Illustrate mathematical relationships using diagrams, two-way tables, graphs, flowcharts, and formulas ❑ Apply assumptions to make a problem simpler ❑ Check to see if an answer makes sense within the context of a situation and change a model when necessary

Grade 3 Common Core State Standards for Mathematical Practice

Student Friendly Version of Standards for Mathematical Practices (SMP)

Standards	Explanations and Examples	Possible Questions and Prompts:	Student "Look Fors"
5. Use appropriate tools strategically.	Mathematically proficient third grade students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.	• What tools will you need? • Will a calculator help? • Will pencil and paper help? Will using a number line, table, diagram, or picture help? • What strategies will you use?	❑ Choose tools that are appropriate for the task. Examples: *Manipulative, Calculator, Digital Technology, Ruler* ❑ Use technological tools to visualize the results of assumptions, explore consequences, and compare predications with data ❑ Identify relevant external math resources (digital content on a website) and use them to pose or solve problems
6. Attend to precision.	Mathematically proficient third grade students develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.	• Will you solve the problem mentally or with pencil and paper? Will using a number line, table, diagram, or picture help? • What do you think the answer or result will be? • What does your answer mean in the context of the problem? • Can you guess and check? • Have you compared your work with anyone else? • Can you represent the definition or rule?	❑ Communicate precisely using appropriate terminology ❑ Specify units of measure and provide accurate labels on graphs ❑ Express numerical answers with appropriate degree of precision ❑ Provide carefully formulated explanations
7. Look for and make use of structure.	In third grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties)	• What relevant information in the problem shows you what relationship (i.e. change, group, compare, ratio, or proportion problem) exists between the elements or parts of the problem? • How do you know that your rule or equation always works? • Are you actively comparing, reflecting on, and discussing multiple solution methods?	❑ Look for a pattern or structure, recognizing that quantities can be represented in different ways ❑ Use knowledge of properties to efficiently solve problems ❑ View complicated quantities both as single objects or compositions of several objects
8. Look for and express regularity in repeated reasoning.	Mathematically proficient third grade students should notice repetitive actions in computations and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"	• What pattern(s) do you notice? How would you describe the pattern(s)? • What calculations, patterns, or principles are repeated? • What mathematical principles will help you in solving the problem? • What if you started with … rather than…? What if you can only use…? • What are the big ideas or key points in this lesson?	❑ Notice repeated calculations and look for general methods and shortcuts ❑ Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and making generalizations

Universal Design for Learning

Prince George's County Public Schools UDL Website

Universal Design for Learning (UDL): is a flexible approach to curriculum planning and implementation that offers all students full and equal opportunities to learn. The UDL framework encourages creating flexible designs from the start that have customizable options, which allow all learners to progress from where they are and not where we would have imagined them to be. Grounded in brain-based research on the diverse ways people learn, UDL offers a practical framework to provide students and teachers with multiple options for learning. The options for accomplishing this are varied and robust enough to provide effective instruction to all learners. Three primary principles, which are based on neuroscience research, guide UDL and provide the underlying framework for the Guidelines.

The Three Principles of UDL		
I. Representation The ***"What"*** of Learning • How does the lesson or task present information and content in different ways? • How do students gather facts and categorize what they see, hear, and read? • How are they identifying symbols, words, and structures?	II. Expression The ***"How"*** of Learning • How does the lesson or task differentiate the ways that student can express what they know? • How do they plan and perform tasks? • How do students organize and express their ideas?	I. Engagement The ***"Why"*** of Learning • How does the lesson or task stimulate interest and motivation for learning? • How do students get engaged? • How are they challenged, motivated, or interested?
1. Perception	1. Physical Action	1. Recruiting Interest
2. Language	2. Expressive Skills and Fluency	2. Sustaining Effort and Persistence
3. Comprehension	3. Executive Function	3. Self-Regulation

Instructional Supports for Learning

UDL Guidelines – Educator Worksheet

**Follow the link for each item below for examples and clarification.*

I. Provide Multiple Means of Representation:	Your notes
1. Provide options for perception	
1.1 Offer ways of customizing the display of information	
1.2 Offer alternatives for auditory information	
1.3 Offer alternatives for visual information	
2. Provide options for language, mathematical expressions, and symbols	
2.1 Clarify vocabulary and symbols	
2.2 Clarify syntax and structure	
2.3 Support decoding of text, mathematical notation, and symbols	
2.4 Promote understanding across language	
2.5 Illustrate through multiple media	
3. Provide options for comprehension	
3.1 Activate or supply background knowledge	
3.2 Highlight patterns, critical features, big ideas, and relationships	
3.3 Guide information processing, visualization, and manipulation	
3.4 Maximize transfer and generalization	
II. Provide Multiple Means for Action and Expression:	Your notes
4. Provide options for physical action	
4.1 Vary the methods for response and navigation	
4.2 Optimize access to tools and assistive technologies	
5. Provide options for expression and communication	
5.1 Use multiple media for communication	
5.2 Use multiple tools for construction and composition	
5.3 Build fluencies with graduated levels of support for practice and performance	
6. Provide options for executive functions	
6.1 Guide appropriate goal setting	
6.2 Support planning and strategy development	
6.3 Facilitate managing information and resources	
6.4 Enhance capacity for monitoring progress	

III. Provide Multiple Means for Engagement:	Your notes
7. Provide options for recruiting interest	
7.1 Optimize individual choice and autonomy	
7.2 Optimize relevance, value, and authenticity	
7.3 Minimize threats and distractions	
8. Provide options for sustaining effort and persistence	
8.1 Heighten salience of goals and objectives	
8.2 Vary demands and resources to optimize challenge	
8.3 Foster collaboration and community	
8.4 Increase mastery-oriented feedback	
9. Provide options for self-regulation	
9.1 Promote expectations and beliefs that optimize motivation	
9.2 Facilitate personal coping skills and strategies	
9.3 Develop self-assessment and reflection	

Grade 3 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. *The Critical Areas for third grade can be found on page 21 in the Common Core State Standards for Mathematics.*

1. **Developing understanding of multiplication and division and strategies for multiplication and division within 100.**
   Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. **Developing understanding of fractions, especially unit fractions (fractions with numerator 1).**
   Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, ½ of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. **Developing understanding of the structure of rectangular arrays and of area.**
   Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sided of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. **Describing and analyzing two-dimensional shapes.**
   Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit faction of the whole.

Grade 3: Year at a Glance (UNITS by Quarter)							
Quarter 1 (45 days) **Focus: Place Value, Properties of Operations and Computation**		**Quarter 2 (43 days)** **Focus: Computation and Introduction to Fraction**		**Quarter 3 (45 days)** **Focus: Fraction and Computation**		**Quarter 4 (47 days)** **Focus: Fractions, Geometry, and Measurement/Data**	
Content Standard	**Suggested Days**	**Content Standard**	**Suggested Days**	**Content Standard**	**Suggested Days**	**Content Standard**	**Suggested Days**
Unit 1 **Place Value, Measurement, Computation** **3.NBT.**1, 2 **3.MD.**1	**14**	**Unit 4** **Measurement, Computation** **3.MD.**5, 5a, 5b, 6, 8 **3.NBT.**2	**15**	**Unit 7** **Fractions** **3.NF.**3, 3a, 3b, 3c, 3d **3.MD.**4	**16**	**Unit 10** **Fraction, Measurement** **3.NF.**2a, 2b, 3a 3b, 3c, 3d **3.MD.**4	**17**
Unit 2 **Properties of Operations, Computation** **3.OA.**1, 2, 3, 4,	**15**	**Unit 5** **Measurement, Place Value, Computation** **3.MD.**7, 7a, 7b 7c, 7d **3.NBT.**3	**14**	**Unit 8** **Measurement, Computation** **3.MD.**1, 2, **3.NBT.**2, 3	**15**	**Unit 11** **Geometry, Measurement/ Data** **3.G.**1, 2 **3.MD.**1, 2 3, 8	**15**
Unit 3 **Properties of Operations, Computation,** **3.OA.**5, 6, 7 8, 9	**16**	**Unit 6** **Fractions, Geometry** **3.NF.**1, 2, 2a, 2b **3.G.**2	**14**	**Unit 9** **Properties of Operations, Computation** **3.OA.**5, 7, 8, 9	**14**	**Unit 12** **Measurement** **3.MD.**5, 5a, 5b, 6, 7, 7c, 7d	**15**

Fluency Expectations:

- **Students fluently multiply and divide within 100. By the end of grade 3, they know all products of two one-digit numbers from memory.**
- **Students fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.**

QUARTER 1

Grade 3, Quarter 1(Suggested Days: 14)
Unit 1: Place Value, Measurement, Computation

Common Core Standards:

- **3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100.
- **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/orthe relationship between addition and subtraction.
- **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Standards for Mathematical Practice:
These Standards represent behaviors that should be embedded within mathematics instruction.

Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*)
(1) Make sense of problems and persevere in solving them
(6) Attend to precision

Reasoning and Explaining *(Math Talk)*
(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others

Modeling and Using Tools (Math Drawings)
(4) Model with mathematics
(5) Use appropriate tools strategically

Seeing Structure and Generalizing *(Math Structure)*
(7) Look for and make use of structure
(8) Look for and express regularity in repeated reasoning

Mathematical Language:
Terms do not represent an exclusive list and should not be taught in isolation.

- Addend
- Algorithm
- AM
- Analog Clock
- Associative Property of Addition
- Benchmark Number
- Commutative Property of Addition
- Difference
- Digit
- Digital Clock
- Elapsed time
- Estimate
- Expanded Form
- Hour
- Hundreds
- Intervals
- Identity Property of Addition
- Minuend
- Minute
- Number Line
- Ones
- Place Value
- PM
- Properties of Operations
- Rounding
- Second
- Standard Form
- Subtrahend
- Sum
- Tens

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- Place value is critical when operating with numbers.
- Place value can be used to name numbers in different ways.
- Place value can be used to compare whole numbers.
- Using place value builds understanding when regrouping is necessary.
- Two numbers can be added in any order.
- Estimation helps us to determine reasonableness of answers.
- The relationships among the operations and their properties promote computational fluency.

- Time can be expressed using different units that are related to each other.
- The duration of an event is called elapsed time and it can be measured.
- Subtraction takes from the original amount.

Essential Questions:

A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- What does it mean to tell time to the nearest minute?
- How do units within a system relate to each other?
- What connections can I make between a clock and a number line?
- How can I use what I know about number lines to help me figure out how much time has passed between two events?
- How can we determine the amount of time that passes between two events?
- Why is understanding place value important?
- How can whole numbers be compared and ordered?
- How are addition and subtraction alike? Different?
- What strategies can be used to find sums and differences
- How does rounding a number change its value relative to other numbers?
- How are digits in a number related?
- How can sums and differences be estimated?
- What makes an estimate reasonable?
- Why do we need mathematical operations?
- How do we know when an estimate or an exact answer is appropriate?

Prior Knowledge:

- Place value to the 1000
- Place value refers to what a digit is worth in a number.
- Each place in a number is worth 10 times more than the place to the right of it
- Standard and expanded forms of numbers
- Addition to 3-digit numbers
- Subtraction to 3-digit numbers
- Fluency of basic addition and subtraction facts.
- Addition and subtraction properties
- Students in second grade learned to tell time to the nearest five minutes.
- Tell time to the hour, half hour and quarter hour
- Distinguish between a.m. and p.m.
- Use the language “quarter to”, “quarter after”, “fifteen minutes after”, and “fifteen minutes before”.
- Tell and write time in hours and half-hours using analog and digital clocks.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• The use of terms “round up” and “round down” confuses many students. For example, the number 39 would round to 40 or it “rounds up”. The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 34 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. Misconceptions occur when emphasis is placed on applying a series of steps, procedures, or rules to round to a specific place value. These steps do not reinforce number sense.	• To remedy this misconception, present situations in context. For example, if it took your mother 58 minutes to prepare dinner last night, you might say that it took her about an hour to prepare dinner. Students need to use a number line labeled in different ways to visualize the placement of the number. Helping students to determine halfway point between two numbers is a more effective way to work with rounding concepts.
• Some students do not understand why subtraction is not commutative. This is important to understanding the addition operation.	• Addition is commutative. The order does not change the sum. 556 + 323 = 323 + 556 Without an understanding of why addition is commutative, students overgeneralize and begin thinking that subtraction is also commutative. It’s important for students to understand that subtraction is finding the difference or distance between two quantities. Subtraction is not commutative. Teachers should not say: “You can’t subtract 500 from 300.” As students’ progress through grade levels, they will subtract these values when they begin their work with negative numbers.
• The hour hand and minute hand on analog clocks are interchangeable.	• Provide opportunities for students to explore and explain what each hand on the clock represents.

Instructional Notes:

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** website.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulative, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.

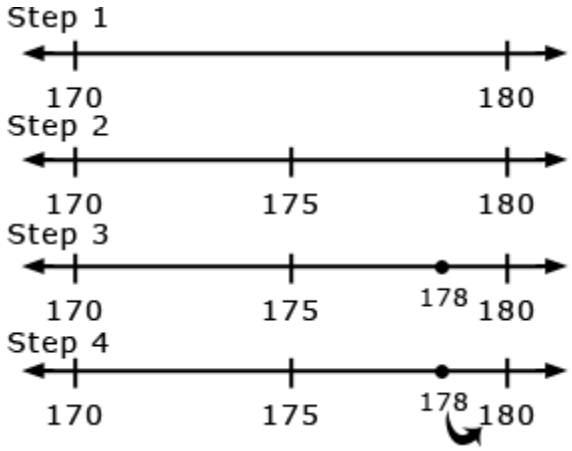
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.

  Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access this document from the Elementary Mathematics Google site.

- **3.NBT.1** This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is for students to have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

  Students learn when and **why** to round numbers. They identify possible answers and halfway points. Then, they narrow where the given number falls between the possible answers and halfway points. They also understand that if a number is exactly at the halfway point of the two possible answers, at this level the number is rounded up.

  Example: Round 178 to the nearest 10.



  Step 1: The answer is either 170 or 180.

  Step 2: The halfway point is 175.

  Step 3: 178 is between 175 and 180.

  Step 4: Therefore, the rounded number is 180.

- **3.NBT.2** This standard refers to "fluently", which means accuracy, efficiency (using a reasonable number of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard or traditional algorithm. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.

  Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

  Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Example 1: Sharon read 573 pages during her summer reading challenge. She was only required to read 399 pages. How many extra pages did Sharon read?

Students may use several approaches to solve the problem including the traditional algorithm.

Other methods students may use are listed below:

399 + 1 = 400, 400 + 100 = 500, 500 + 73 = 573, therefore 1+ 100 + 73 = 174 pages (Adding up strategy)
400 + 100 is 500; 500 + 73 is 573; 100 + 73 is 173 plus 1 (for 399, to 400) is 174 (Compensating strategy)
Take away 73 from 573 to get to 500, take away 100 to get to 400, and take away 1 to get to 399.
Then 73 +100 + 1 = 174 (Subtracting to count down strategy)
399 + 1 is 400, 500 (that's 100 more). 510, 520, 530, 540, 550, 560, 570, (that's 70 more), 571, 572, 573 (that's 3 more) so the total is 1 + 100 + 70 + 3 = 174 (Adding by tens or hundreds strategy)

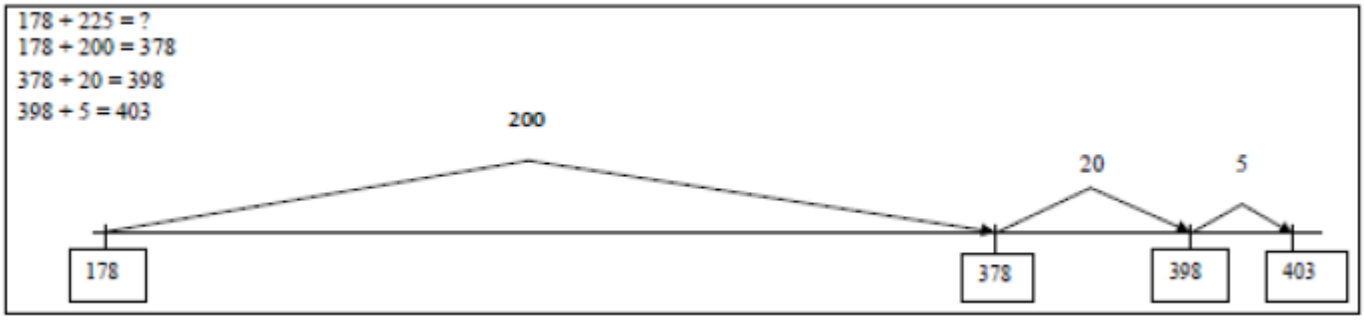
Example 2: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1
100 + 200 = 300
70 + 20 = 90
8 + 5 = 13
300 + 90 + 13 = 403 students

Student 2
I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.

Student 3
I know 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So, I have 4 hundreds plus 3 more which is 403.

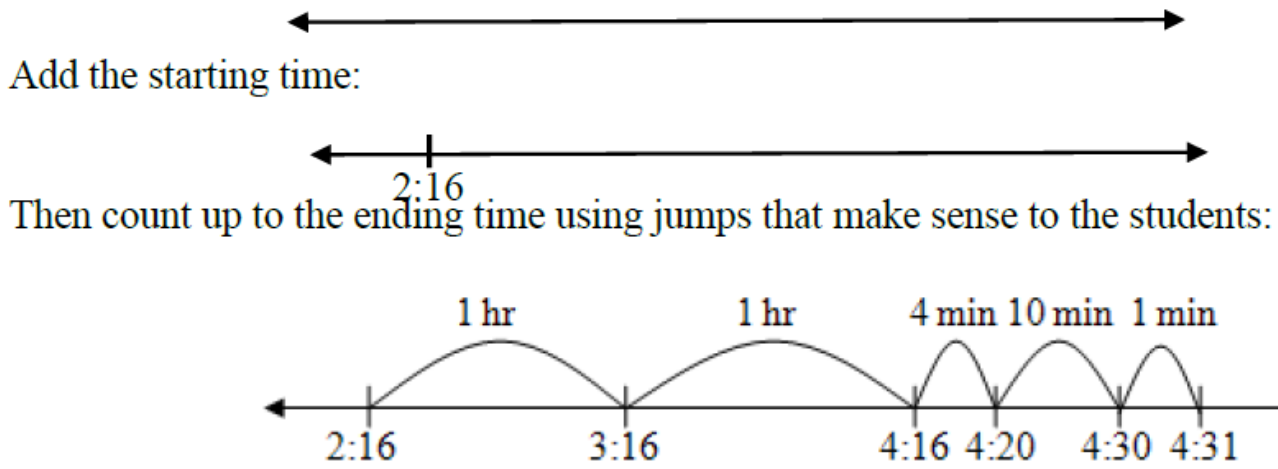
Student 4



- **3.MD.1** This standard requires students to solve problems involving elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students.

  Provide opportunities for students to explore linear models of time as well as a traditional analog or digital clock. The linear model can be created using an open number line.

Example: To find the elapsed time from 2:16 pm to 4:31 pm, start with an open number line:



Instructional Resources:

Manipulatives:

- Analog Clock
- Base Ten Blocks
- Digital Clock
- Hundreds Chart
- Number Line
- Place Value Mat

Instructional Supplement *(Refer to the Elementary Mathematics Google site):*

- Lesson Seed: #1
- Instructional Strategies: #1-2, #5, #8, #10, #24 - #25, #59
- Extensions for Advanced Learners B: Field Day
  Time Zones
  Zoomation!

Literature Connection:

- How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000? by Helen Nolan
- The Grapes of Math by Greg Tang
- Lucky Beans by Becky Bertha
- Math Potatoes by Greg Tang
- One Hundred Shoes: A Math Reader (Step-into-Reading, Step 2) by Charles Ghigna
- Telling Time: How to Tell Time on Digital and Analog Clocks! by Jules Older
- Kids Time With Father Time-Learn How To Tell Time The Time With Father Time by Daisy Wright
- Animal Word Problems Starring Addition and Subtraction (Math Word Problems Solved) by Rebecca Wingard-Nelson
- Addition the Fun Way Book for Kids: A Picture Method of Learning the Addition Facts by Judy Liautaud
- Domino Addition by Lynette Long
- Count On Clifford by Norman Bridwell

	• Animals on Board by Stuart J. Murphy • My Rows and Piles of Coins by Tololwa M. Mollel **Web Links:** • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Rounding Numbers http://www.wartgames.com/themes/math/rounding.html • Mental Math Strategies http://olc.spsd.sk.ca/de/math1-3/p-mentalmath.html • Place Value and Rounding http://www.funbrain.com/tens/index.html • Math Games for Rounding and Place Value http://www.ehow.com/way_5182955_math-games-rounding.html http://www.wartgames.com/themes/math/rounding.html • National Library of Virtual Manipulatives - Interactive Glossary http://nlvm.usu.edu/en/nav/vlibrary.html • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • K-5 Math Teaching Resources – Math Games and Hands-on Activities http://www.k-5mathteachingresources.com/ • The Teaching Channel: Lesson Ideas and Videos https://www.teachingchannel.org/ • Math Games and Activities http://illuminations.nctm.org/ActivitySearch.aspx • Tasks and Assessment Tools http://insidemathematics.org/index.php/mathematical-content-standards • Interactive Glossary http://www.ronblond.com/MathGlossary/ • Place Value, Number Line, Measurement http://www.mathsframe.co.uk/resources/category/Partitioning_and_Place_Value.aspx

Grade 3, Quarter 1(Suggested Days: 15)
Unit 2: Properties of Operations, Computation

Common Core Standards:

- **3.OA.1** Interpret products of whole numbers, (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.)*For example, describe a context in which a total number of objects can be expressed as 5 x 7.*
- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = __ ÷ 3, 6 x 6 =?*

Standards for Mathematical Practice: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Area Model • Array • Column • Decompose • Decrease • Digit • Dividend • Division • Divisor • Equal Groups/Parts • Equation • Expression • Equivalence • Factor • Increase • Inverse operations • Multiplication • Partition • Product • Properties of Operations • Quotient • Row • Solve

Enduring Understandings (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- Multiplication and division are inverses; they undo each other.
- Rectangular arrays can be decomposed into rows or columns.
- Multiplication and division situations involve equal-size groups, arrays, and/or area models.
- With whole number ass the divisor increases, the quotient decreases.
- An equal sign to represents balance or equivalence.
- Flexible methods of computation for all operations involve composing an decomposing numbers in a variety of ways.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How are multiplication and division related?
- What strategies can be used to solve multiplication or division problems?
- How can multiplication help you to divide?
- How can we model multiplication?
- How can the same array represent both multiplication and division?
- What are ways I can multiply?
- What is the relationship between products and sums, quotients and differences?
- How can we write equations to represent multiplication or division models created?
- Is there more than one way to multiply or divide a number to get the same product or quotient?
- What do the parts of a division problem represent?
- What is the relationship between the divisor and the quotient?

Prior Knowledge:

- Represent and solve problems using addition and subtraction.
- Use addition and subtraction within 100.
- Fluently add and subtract within 100.
- Use addition and subtraction to solve one-and two-step word problems.
- Explain why addition and subtractions strategies work.
- Understand the meaning of the equal sign.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students think that division is commutative.	• Students should use manipulatives to create division situations in order to see that division is not commutative. **Example:** There are 5 candy bars on the table. If there are 25 students in Tony's class, how many candy bars will each student receive?
• Students do not understand or confuse the meaning of the multiplication symbol.	• By providing opportunities for students to engage in contextual situations involving multiplication will help students understand that the multiplication symbol "x" means 'groups of' and therefore understand that 4 x 5 means four groups of 5 objects.
• A common misconception for student is understanding the equal sign. They do not understand that the equal sign represent a balance or equivalence.	• Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown indifferent positions.

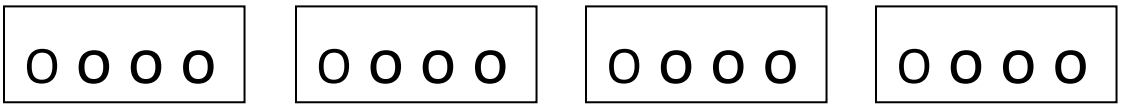
Instructional Notes:

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** website.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access this document from the Elementary Mathematics Google site.
- Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of objects rather than individual objects. Students should learn that the multiplication symbol "x" means "groups of" and problems such **as 6 x 8 refer to 6 groups of 8 objects.**
- Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.
- Researchers and mathematics educators advise against providing **"key words"** for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it.

- **3.OA.2** This standard focuses on two distinct models of division: **partition models** and **measurement (repeated subtraction) models.**

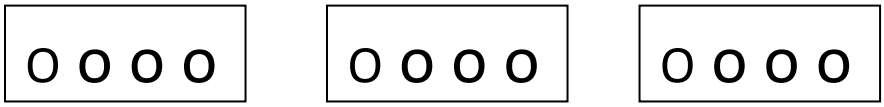
  Partition models provide students with a total number and the number of groups**.** These models focus on the question, **"How many objects are in each group so that the groups are equal?"**

  Example: Denise place 16 pieces of candy on the table. If she is sharing the candy equally among four bags, how many pieces of candy will go into each bag?



  Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, **"How many equal groups or equal shares can you make?"**

  Example 1: There are 12 pieces of candy on the table. If you put 4 pieces in each bag, how many bags will you need?



  Example 2: Harry the monkey loves bananas. Joseph, his trainer, has 28 bananas. If he gives Harry 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
28	28 – 4 = 24	24 – 4 = 20	20 – 4 = 16	16 – 4 = 12	12 – 4 = 8	8 – 4 = 4	4 – 4 = 0

  Solution: The bananas will last 7 days.

- **3.OA.3** This standard addresses various problem solving context and strategies that students are expected to use while solving word problems involving multiplication and division.

  Students should use a variety of representations for creating and solving one and two-step word problems.

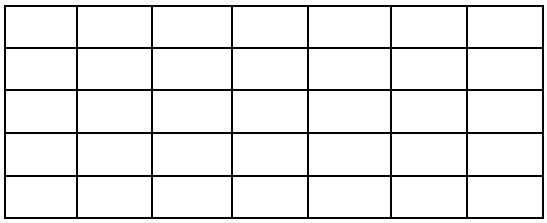
  Examples of Multiplication:

  1. Glen divides 3 packs of 8 cookies among 4 people, how many cookies does each person receive? *(3 x 8 = 24, 24 ÷ 4 = 6).*

2. There are 35 desks in the classroom. If the teacher puts 7 desks in each row, how many rows are there?

This task can be solved using an <u>array model</u>.

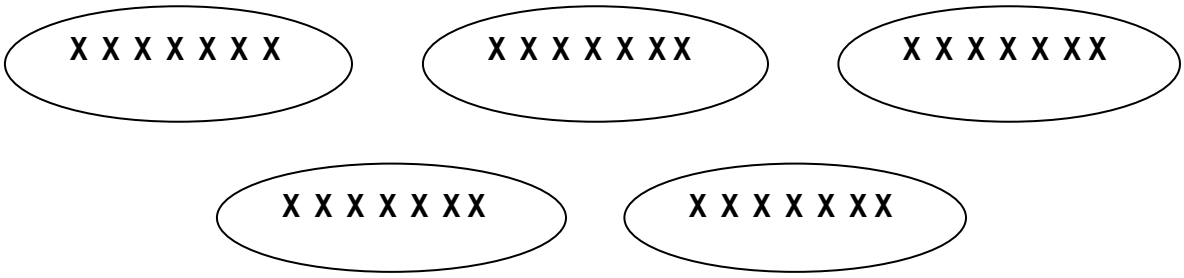
7 desks in a row.



Solution: 5 rows

This task can also be solved by <u>drawing pictures of equal groups.</u>

5 groups of 7 equals 35 objects

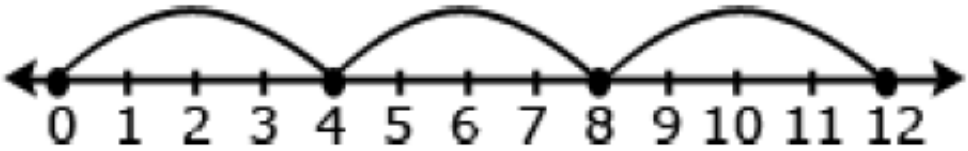


A student can <u>reason through the problem mentally or verbally</u>.

"I know 7 and 7 equal 14. 14 and 14 equal 28. 28 and 7 equal 35. Therefore, there are 5 groups of 7 giving a total of 35 desks in the classroom."

3: A **<u>number line</u>** can also be used to demonstrate multiplication by showing equal jumps.

There are 12 pieces of candy on the table. If you put 4 pieces in each bag, how many bags will you need?

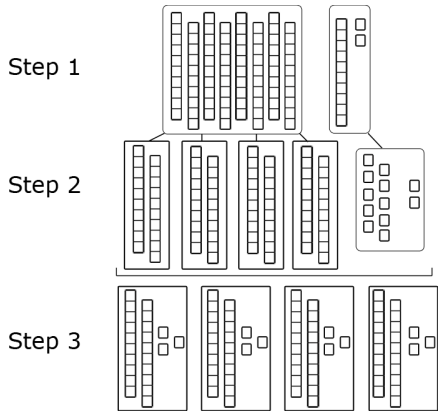


Examples of Division:

Determining the number of objects in each share (**partition model of division**, where the size of the groups is unknown):

1. There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class.
   (☐ ÷ 4 = 6. There are 24 students in the class).

2. The bag has 92 pencils. Siyan and her three friends want to share them equally. How many pencils will each person receive?



Solution: Each person will receive 23 pencils.

Instructional Resources:

Manipulatives:

- Base Ten Blocks
- Connect Cubes
- Craft Sticks
- Grid Paper
- Hundreds Chart
- Number Line
- Objects to Represent Groups: Beans, Pasta, Coins, etc
- Part-Part Whole Mats
- Two Colored Counters
- Unifix Cubes

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Lesson Seed: #2
- Instructional Strategies: #33, #38, #51 - #52
- Extensions For Advanced Learners B: Streets and Avenues
  Multiplication Bingo
  Rap Time
- Extensions For Advanced Learners D: Batting Average
- MSDE Unit 3: Represent and Solve Problems Involving Multiplication & Division

- MSDE Lesson Plan: Use Multiplication and Division with 100 to Solve Word Problems and to Determine the Unknown

- MSDE Lesson Seeds: Fluency Through I Have, Who Has
  Fluency Through Salute Game

Literature Connection:

- Amanda Bean's Amazing Dream by Cindy Neuschwander
- The Best of Times by Gregory Tang
- Cheetah Math by Ann Whitehead Nagda
- A Remainder of One by Elinor J. Pinczes
- Great Estimations by Bruce Goldstone
- Times Tables the Fun Way: Book for Kids: A Picture Method of Learning the Multiplication Facts by Judy Liautaud
- Multiplying Menace: The Revenge Of Rumpelstiltskin (A Math Adventure) by Pam Calvert
- Divide and Ride (MathStart 3) by Stuart J. Murphy
- 2 X 2 = Boo: A Set of Spooky Multiplication Stories by Loreen Leedy
- One Grain Of Rice: A Mathematical Folktale by Demi
- Anno's Mysterious Multiplying Jar by Masaichiro Anno
- One Hundred Hungry Ants by Elinor J Pinczes
- Each Orange Had 8 Slices by Paul Giganti
- Big Truck and Car Word Problems Starring Multiplication and Division (Math Word Problems Solved) by Rebecca Wingard-Nelson
- More M&M's Brand Chocolate Candies Math by Barbara Barbieri McGrath

Web Links:

- Elementary Mathematics Google Site
  https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/

- PGCPS Universal Design for Learning (UDL)
  http://www1.pgcps.org/UDL/index.aspx?id=127354.

- MSDE Common Core Curriculum Framework
  http://www.mdk12.org/instruction/commoncore/index.html

- Common Core State Standards
  http://www.corestandards.org/

- Base Ten Blocks and Algorithms
  http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm#div

- Math Glossary
  http://math.about.com/library/ble.htm

- Mental Math Strategies
  http://olc.spsd.sk.ca/de/math1-3/p-mentalmath.html

	• Interactive Activities http://www.pbs.org/teachers/math/ • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • National Library of Virtual Manipulatives - Interactive Glossary http://nlvm.usu.edu/en/nav/vlibrary.html • Word Problem Strategies http://www.mathstories.com/strategies.htm • Multiplication facts http://www.studyzone.org/testprep/math4/e/multiplicationstrategies3l.cfm • Division Strategies http://www.youtube.com/watch?v=_RM8yaqvG8c • Multi-step word problems http://www.mathplayground.com/wordproblems.html • Games and Activities http://mathwire.com/ • K-5 Math Teaching Resources – Math Games and Hands-on Activities http://www.k-5mathteachingresources.com/ • Tasks and Assessment Tools http://insidemathematics.org/index.php/mathematical-content-standards • Interactive Glossary http://www.ronblond.com/MathGlossary/

Grade 3, Quarter 1(Suggested Days: 16)
Unit 3: Properties of Operations, Computation

Common Core Standards:

- **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property.)*
- **3.OA.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
- **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ *45 = 8) or properties or operations. By the end of Grade 3, know from memory all products of two one-digit numbers.*
- **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Standards for Mathematical Practice:
These Standards represent behaviors that should be embedded within mathematics instruction.

Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*)
(1) Make sense of problems and persevere in solving them
(6) Attend to precision

Reasoning and Explaining *(Math Talk)*
(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others

Modeling and Using Tools (Math Drawings)
(4) Model with mathematics
(5) Use appropriate tools strategically

Seeing Structure and Generalizing *(Math Structure)*
(7) Look for and make use of structure
(8) Look for and make use of structure

Mathematical Language:
Terms do not represent an exclusive list and should not be taught in isolation.

- Addend
- Addition
- Arithmetic Patterns
- Associative Property
- Commutative Property
- Composing
- Counters
- Decomposing
- Difference
- Distributive Property
- Divide
- Division
- Equation
- Factor
- Fluently
- Four Operations
- Identity Property of Multiplication
- Mental Computation
- Minuend
- Multiplication
- Multiply
- Order of Operations
- Properties of Operations
- Relationship
- Reasonableness
- Rounding
- Strategies
- Subtraction
- Subtrahend
- Sum
- Unknown Factor
- Unknown Quantity
- Variable
- Zero Property of Multiplication

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- When adding or multiplying two numbers, the order in which the two numbers are added or multiplied does not change the sum or product (Commutative Property).
- You can add or multiply numbers regardless of how they are grouped (Associative Property).
- Multiplying a sum by a number is the same as multiplying each addend by the number and then adding the products (Distributive Property).
- Through the properties of operations (commutative, associative, distributive), we understand the relationships of various mathematical functions.
- The commutative, associative, and distributive properties can be used to develop efficient strategies to multiply.
- Repeated addition involves joining equal groups and is one way to think about multiplication.
- Addition, subtraction, multiplication, and division operate under the same properties in algebra as they do in arithmetic.

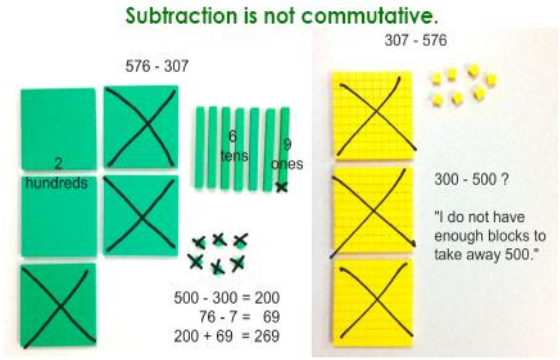
Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How is the commutative property of multiplication evident in an array model?
- How does understanding the commutative property help us create arrays?
- How does drawing an array help us think about different ways to decompose a number?
- What is the relationship between products and sums, quotients and differences?
- What are some strategies for solving for unknowns in equations?
- How can we determine numbers that are missing on a multiplication table by knowing multiplication patterns?
- How does mental math help you calculate more quickly and develop deeper understanding of numbers?
- What strategies can be used to find sums and differences?
- How do we represent currency and add or subtract money amounts?
- What is the relationship between products and sums, quotients and differences?

Prior Knowledge:

- Composing and decomposing whole numbers.
- Knowledge of skip counting and explain “why” the pattern works.
- Use addition and subtraction within 100.
- Use addition and subtraction to solve one-and two-step word problems.
- Explain why addition and subtractions strategies work.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students think that there is only one way to compose or decompose a number. **For example,** they are unable to see 347 as 34 tens and 7 ones and only see it as 3 hundreds, 4 tens and 7 ones.	• Provide situations for students to decompose numbers in various ways without using the highest place value.

• Students think that subtraction is also commutative as addition. They think that 576 - 307 is the same as 307 - 576.	• It's important for students to understand that subtraction is finding the difference or distance between two quantities. Use of place value blocks can clear this misconception.  Remember **not** to say, "You can't subtract 500 from 300." As student progress through the grade levels, they will subtract these values when they begin their work with negative numbers.

Instructional Notes:

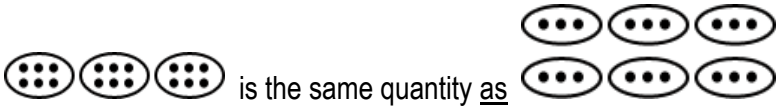
- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL).**
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulativse, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access this document from the Elementary Mathematics Google site.
- Researchers and mathematics educators advise against providing **"key words"** for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it.
- 3.OA.5 Students should represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties of operations (associative, commutative, and distributive). They should multiply by 1 and 0 and divide by 1. They should change the order of numbers to determine that the order of numbers does not make a difference in multiplication but does make a difference in division.

Provide opportunities for students to use the **associative property** (grouping property), which states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies

6 x 4 x 2, a student could rearrange the numbers to first multiply 4 x 2 = 8 and then multiply 8 x 6 = 48.
Provide experiences for students to use the **commutative property** (order property), which states that the order of numbers does not matter when adding, or multiplying numbers. For example, if a student knows that 6 x 5 = 30, then they also know that 5 x 6 = 30. The array below could be described as 5 x 6 or 6 x 5. Students should have flexibility in being able to describe both dimensions of an array.

Example 1: 3 x 6 = 6 x 3

In the following diagram it may not be obvious that 3 groups of 6 is the same as 6 groups of 3. A student may need to count to verify this.



Example 2: 4 x 3 = 3 x 4

An array explicitly demonstrates the concept of the commutative property.

4 rows of 3 or 4 x 3 3 rows of 4 or 3 x 4

Splitting arrays can help students understand the **distributive property** of multiplication as a strategy for using products they know to solve for products they don't know.

Example: Students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.

```
O O O O O O O O O     O O O O O     O O O O
O O O O O O O O O     O O O O O     O O O O
O O O O O O O O O  =  O O O O O  +  O O O O
O O O O O O O O O     O O O O O     O O O O
O O O O O O O O O     O O O O O     O O O O
O O O O O O O O O     O O O O O     O O O O
```

6 x 5 = 30 **6 x 4 = 24** **Sum: 30 + 24 = 54**

- **3.OA.7** By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts.

  Strategies students may use to attain fluency include:

  - Multiplication by zeros and ones
  - Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
  - Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
  - Five facts (half of tens)
  - Skip counting (counting groups of __ and knowing how many groups have been counted)
  - Square numbers (ex: 3 x 3)
  - Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
  - Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
  - Turn-around facts (Commutative Property)
  - Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
  - Missing factors (Ex. 3 x ? = 21)

- **3.OA.8** This standard calls for students to represent problems using equations with a letter to represent unknown quantities.

  Example: Branden runs 3 miles a day. His goal is to run 27 miles. After 4 days, how many miles does Branden have left to run in order to meet his goal? Write an equation and find the solution: **(3 x 4 + a = 27).**

  This standard also refers to estimation strategies, including using compatible numbers (numbers with sum to 10, 50, or 100) or rounding. The focus is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

  Example: On their vacation, the Soungie family traveled 277 miles on the first day, 188 miles on the second day and 24 miles on the third day. What is the total number of miles they traveled?

  Some Estimation Strategies for the Problem:

  Strategy A

  I first thought about 277 and 24. I noticed that their sum is about 300. Then I knew that 188 is close to 200. When I add 300 and 200, I get 500.

  Strategy B

  I first thought about 188. It is close to 200. I also have 2 hundreds in 277. That gives me a total of 4 hundreds. Then, there is 77 in 277 and the 24. When I put 77 and 24 together it is close to 100. When I add that hundred to the 4 hundreds that I already have, I end up with 500.

  Strategy C

  I rounded 277 to 300. I rounded 188 to 200. I rounded 24 to 20. When I added 300, 200 and 20, my answer is about 520. Have students discuss the reasonableness of their estimation as it relates the actual answer.

- **3.OA.9** This standard requires students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

  This standard also calls for students to identify patterns relating to the properties of operations.

  Examples:
  - Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).
  - Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
  - On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
  - On an addition chart, the sums in each row and column increase by the same amount.

What do you notice about the numbers highlighted on the multiplication table? Explain a pattern using properties of operations.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Example: 6 x 5 = 30 and 5 x 6 = 30.

When one changes the order of the factors they will still get the same product (commutative property).

Instructional Resources:

Manipulatives:
- Addition Table
- Base Ten Blocks
- Connecting Cubes
- Counters
- Grid/Graph Paper
- Multiplication Table
- Number Line
- Tiles
- Unifix Cubes

Instructional Supplement *(Refer to the Elementary Math Google site):*
- Lesson Seed: #3
- Instructional Strategies: #7, #22, #32, #34, #39 - #41
- Extensions for Advanced Learners A: Numberline Quiz
  Musical Numbers
- Extensions for Advanced Learners D: Food Menu
  Mr. Smith's Orange Orchard
- MSDE Unit 3: Represent and Solve Problems Involving Multiplication & Division
- MSDE Unit 3: Operations and Algebraic Thinking - Solve Problems Involving the Four Operations, and Identify and Explain Patterns in Arithmetic

- MSDE Lesson Plan: Solve two-step Word Problems Involving the Four Operations
  Use Multiplication and Division with 100 to Solve Word Problems and to Determine the Unknown
- MSDE Lesson Seeds: Patterns Involving Multiplication
  Number of the Day Activity
  Patterns on the Hundredths Chart
  Using Estimation and Mental Math

Literature Connection:

- Great Estimations by Bruce Goldstone
- Betcha! Estimating (Mathstart, Level 3 by Stuart L. Murphy)
- Take a Guess: A Look at Estimation (Spyglass Books) by Janine Scott
- The, I Hate Mathematics! by Marilyn Burns
- Safari Park (MathStart 3) by Stuart J. Murphy
- 7 x 9 = Trouble! By Claudia Mills
- Math Curse by Jon Scieszka
- The Greedy Triangle by Marilyn Burns

Web Links:

- Elementary Mathematics Google Site
  https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/
- PGCPS Universal Design for Learning (UDL)
  http://www1.pgcps.org/UDL/index.aspx?id=127354.
- MSDE Common Core Curriculum Framework
  http://www.mdk12.org/instruction/commoncore/index.html
- Common Core State Standards
  http://www.corestandards.org/
- Problem Solving Strategies
  http://www.thesingaporemaths.com/stratf.html
- Word Problem Solving Strategies
  http://library.thinkquest.org/4471/learn.htm
- RICE – Math Problem Solving Strategy
  http://www.mrswhitehead.com/?p=1284
- Multiplication Strategies
  http://www.youtube.com/watch?v=zSHktBF8CjY
- Multiplication Strategies
  http://www.youtube.com/watch?v=Qmn8akImPVQ
- Division Strategies
  http://www.youtube.com/watch?v=4FvbY8avRwM
- Multiplication and Division Pick 'n' Mix 1
  http://www2.nzmaths.co.nz/number/Operating%20Units/MultDivStrategies.aspx

	• Achieve the Score http://www.achievethecore.org/steal-these-tools • Division Strategy http://www.abc123kidz.com/division_strategies.html • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • K-5 Math Teaching Resources – Math Games and Hands-on Activities http://www.k-5mathteachingresources.com/ • The Teaching Channel https://www.teachingchannel.org/

QUARTER 2

Grade 3, Quarter 2 (Suggested Days: 15)
Unit 4: Measurement, Computation

Common Core Standards:

- **3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement.
- **3.MD.5.a.** A square with side length 1 unit, called a "unit square," is said to have "one square unit" of area, and can be used to measure area.
- **3.MD.5.b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
- **3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- **3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
- **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standards for Mathematical Practice:
These Standards represent behaviors that should be embedded within mathematics instruction.

Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*)
(1) Make sense of problems and persevere in solving them
(6) Attend to precision

Reasoning and Explaining *(Math Talk)*
(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others

Modeling and Using Tools (Math Drawings)
(4) Model with mathematics
(5) Use appropriate tools strategically

Seeing Structure and Generalizing *(Math Structure)*
(7) Look for and make use of structure
(8) Look for and express regularity in repeated reasoning

Mathematical Language:
Terms do not represent an exclusive list and should not be taught in isolation.

- Add
- Addend
- Algorithm
- Area
- Attribute
- Centimeter
- Composing
- Customary Unit
- Decomposing
- Difference
- Equation
- Estimate
- Foot
- Gaps
- Minuend
- Overlaps
- Perimeter
- Place Value
- Plane Figure
- Polygon
- Product
- Properties of Operations
- Relationship
- Subtract
- Subtrahend
- Sum
- Square Meter
- Square Inch
- Square Centimeter
- Square Foot
- Strategy
- Unit
- Unit Square
- Unknown Quantity

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- The region inside a shape is its area. It is a two dimensional space.
- Area can be found by counting the square units or by multiplying arrays of square units.
- Area and perimeter are attributes used to describe and measure two-dimensional figures.
- The distance around a polygon is its perimeter, which is the sum of the lengths of the sides.
- Different shapes can have the same perimeter.
- Numbers can be decomposed and recomposed into component parts to add and subtract multi-digit numbers efficiently.
- Linear units are used to measure perimeter.

Essential Questions*:*
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- What strategies will help you add numbers quickly and accurately?
- How does a number's position affect its value?
- How do we choose the appropriate unit of measurement?
- How do you find the perimeter and area of a shape?
- What is the difference between area and perimeter?
- What strategies will help me solve for an unknown side when finding perimeter?
- How can rectangles have the same perimeter but have different areas?

Prior Knowledge:

- Familiar with shapes and their attributes including rectangles, squares, quadrilaterals, pentagons, hexagons, circles, and rectangles.
- Knowledge of multiplication
- Knowledge of Polygons
- Composing and decomposing numbers
- Partial Sums
- Counting on, counting by 5's,10's, and 100's
- Understand that addition and subtraction are inverse operations
- Knowledge of strategies to add and subtract two-digit numbers
- Knowledge of strategies for solving word problems

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students may have misconceptions when working with large numbers. For example, when given 344 + 176, students tend to simply guess by recalling basic facts and /or not paying attention to the place value	• When working with large numbers, students should represent them with models and apply strategies based on place value and the understanding of addition and subtraction.
• Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter.	• Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.
• Students may become confused with place value when adding and subtracting.	• Provide opportunities for students to use place value chart/mat when adding and subtracting.
• Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter.	• Pose problems situations that require students to explain whether they are finding the perimeter or area.

Instructional Notes:

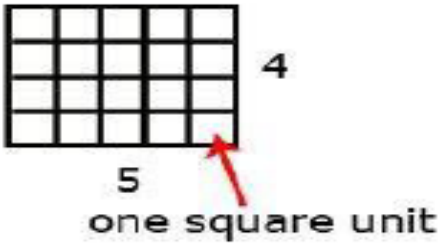
As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access this document from the Elementary Mathematics Google site.

- **3.MD.5 - 6 Students developing understanding of using square units to measure area by:**

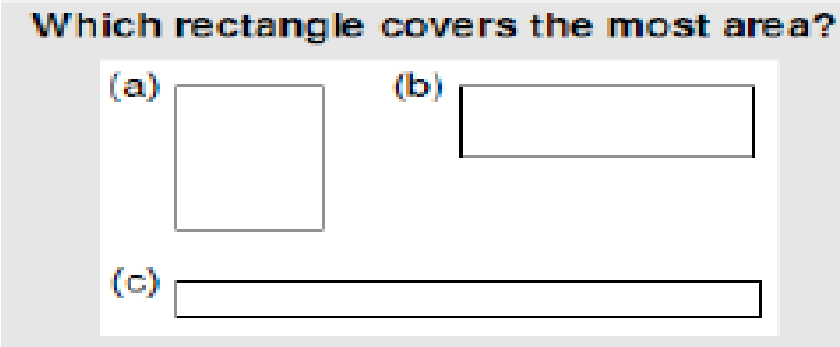
    1. Using different sized square units.

    2. Filling in an area with the same sized square units and counting the number of squares.



Students should cover rectangular shapes with tiles and count the number of units (tiles) to help develop the understanding and meaning that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formula should not be introduced before students discover the meaning of area.

Example: These rectangles are formed by unit squares (tiles). Students are not given the rectangles Dimensions: Figure a: (4 by 3); Figure b (2 by 6); Figure c: (1 row of 12)



<u>Ask questions such as:</u>

What does a column of units in a rectangle represent? (length)

What does a row of units in a rectangle represent? (width)

- **3.MD.8** Students develop an understanding of the concept of perimeter through various experiences, such as walking around the perimeter of a room or using rubber bands to represent the perimeter of a plane figure on a geoboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

  Students should find all the possible rectangles for a given **perimeter** (e.g., find the rectangles with a perimeter of 14 cm.) They record on an organized list or a table, and determine whether they have all the possible rectangles. Following this experience, students can reason about connections between their representations, side lengths, and the perimeter of the rectangles.

  Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given **area** (e.g. find the rectangles that have an area of 12 square units.) They record all possibilities, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

Instructional Tools/Resources:

Manipulatives:

- Base Ten Blocks
- Connecting Cubes
- Counters
- Geoboards
- Grid
- Place Value Chart/Mat
- Rubber Bands
- Tiles

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Lesson Seed: #4
- Instructional Strategies: #18, #20
- Extensions for Advanced Learners A: Rover's All Over
- Extensions for Advanced Learners B: Rectangular Patterns
- Extensions for Advanced Learners C: Tile It
  Frame It

Literature Connection:

- Millions to Measure by David M. Schwartz
- Perimeter, Area, and Volume by David A. Adler
- Spaghetti and Meatballs for All by Marilyn Burns
- Teotihuacan: Designing an Ancient Mexican City: Calculating Perimeters and Areas of Squares and Rectangles by Lynn Georg

	Web Links: • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Mathematics Educator Effectiveness Academy – Unit, Lesson Plans/Seeds http://mdk12.org/instruction/academies/eeaMATH_elementary_ur.html • Junior Architects – Finding Perimeter and Area http://illuminations.nctm.org/LessonDetail.aspx?ID=L651 • Find the Perimeter and Area Playground http://www.mathplayground.com/area_perimeter.html • Perimeter and Area of Polygons http://www.mathgoodies.com/lessons/toc_vol1.html • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • Perimeter and Area http://www.k-5mathteachingresources.com/ • K-5 Math Teaching Resources – Math Games and Hands-on Activities http://www.k-5mathteachingresources.com/ • The Teaching Channel https://www.teachingchannel.org/ • Interactive Lessons to Clarify Standards http://learnzillion.com/common_core/math

Grade 3, Quarter 2 (Suggested Days: 14)
Unit 5: Measurement, Place Value, Computation

Common Core Standards:

- **3.MD.7** Relate area to the operations of multiplication and addition.
- **3.MD.7.a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- **3.MD.7.b**. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- **3.MD.7.c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.
- **3.MD.7.d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
- **3.NBT.3** Multiply one digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Standards for Mathematical Practice: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Addend • Additive • Area • Area Model • Arrays • Decompose • Distributive Property • Digit • Equation • Factor • Gap • Non-Overlapping • Multiply • Overlapping • Place Value • Product • Property of Operations • Rectangle • Rectangular • Rectilinear Figure • Strategies • Sum • Technique • Tiling

Enduring Understandings (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- The area of rectangular objects depends on their dimensions
- Area is a two dimensional measure.
- The space inside a rectangle or square can be measured in square units.
- Area is additive when rectilinear figures are broken into parts.
- The relationships among the operations and their properties promote computational fluency.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- What strategies can be used to solve multiplication problems?
- How can you use the properties of operations to explain or justify answers?
- How can you apply the rules of the distributive property to find the area of a rectilinear figure?
- How can you model multiplication by 10?
- What happens to a number when it is multiplied by ten?

Prior Knowledge:

- Represent and solve problems using addition and subtraction.
- Use place value understanding and the properties of addition to perform multi-digit arithmetic.
- Fluently add and subtract within 100.
- Use addition and subtraction to solve one-and two-step word problems.
- Knowledge of addition and subtraction
- Knowledge of repeated addition and subtraction
- Composing and decomposing numbers

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students tend to confuse perimeter and area. When they count the distance around a shape or figure they think they found the area.	• Provide multiple experiences for students to use manipulatives (i.e. goeboards, grid paper) to find area and perimeter. Pose problems situations that require students to explain whether they are finding the perimeter or area.

Instructional Notes:

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.
- To help students develop the understanding and meaning that area is a measure of covering, provide opportunities for students to cover rectangular shapes with tiles and count the number of units (tiles).
- Provide opportunities for students to investigate arithmetic properties using area models.

  Example: Situations for students to rotate rectangular arrays physically and mentally, understanding that their areas are preserved under rotation. **For example**, 5 x 8 = 8 x 5, illustrating the commutative property of multiplication.

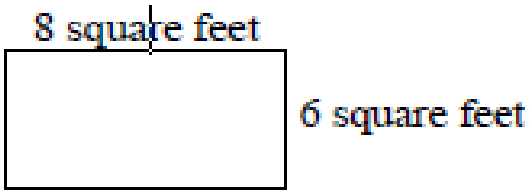
- Students can lay out unit squares and count how many square units it takes to completely cover a rectangle without overlaps or gaps.
- Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.
- A connection needs to be made between the number of squares it takes to cover a rectangle and the dimensions of the rectangle. **Ask questions such as:**
  - What does the length of a rectangle describe about the squares covering it?
  - What does the width of a rectangle describe about the squares covering it?

Students should also make the connection of the area of a rectangle to the area model used to represent multiplication. This connection justifies the formula for the area of a rectangle.

- **3.MD.7.b**

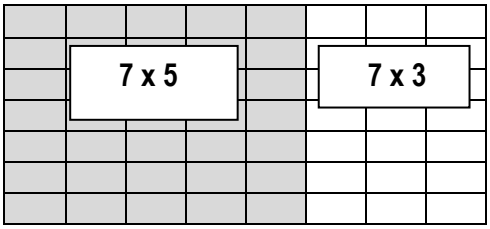
Example:

Mr. Morrison wants to tile his bathroom floor using 1- sq. ft. tiles. How many square foot tiles will he need to cover the entire area of his bathroom floor?

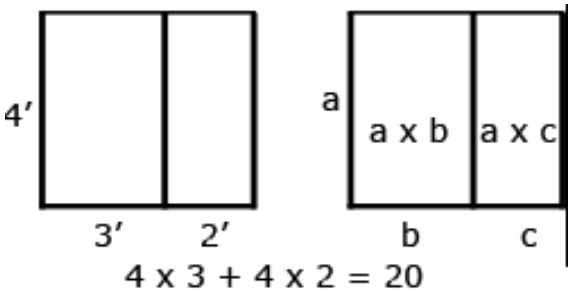


- **3.MD.7, 7c** This standard extends students' work with the distributive property. Students investigate arithmetic properties using area models

Example 1: In the picture below the area of a 7 x 8 figure can be determined by decomposing the 8 into 5 and 3, and reach the answer by multiplying 7 x 5 and 7 x 3 and then add the two products 35 +21 = 56.



Example 2: Sheldon made a poster that was 4' by 3'. Mikale made a poster that was 4' x 2'. They placed their posters on the wall side-by-side so that that there was no space between them. How much area will the two posters cover?

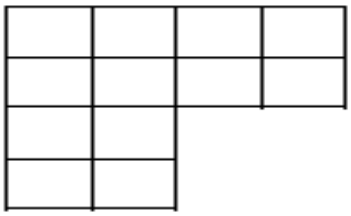


$$4 \times 3 + 4 \times 2 = 20$$
$$4\,(3 + 2) = 20$$
$$4 \times 5 = 20$$

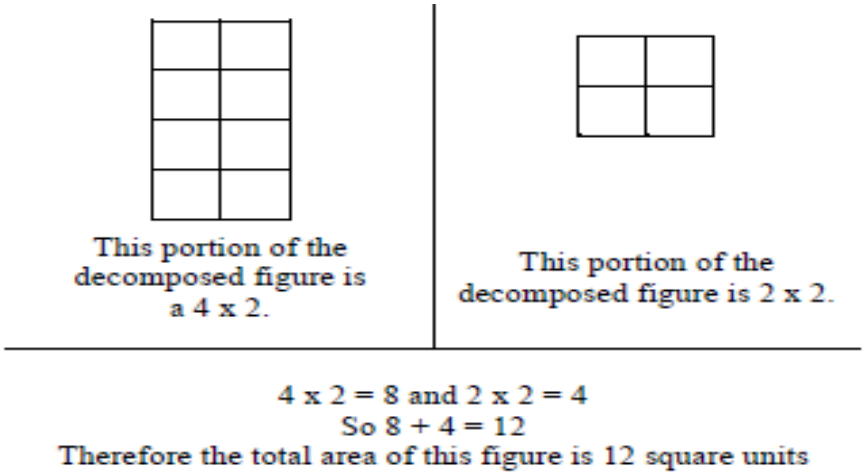
Solution: 20 sq. ft.

- **3.MD.7.d** This standard uses the word rectilinear. **A rectilinear figure** is a polygon that has all right angles.

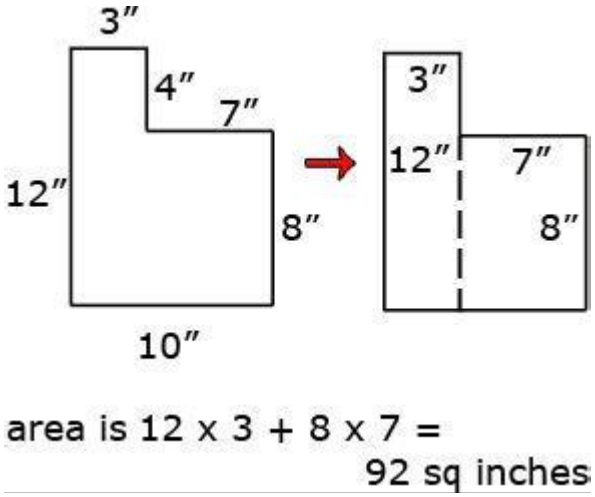
Example 1:



How could this rectilinear figure be decomposed to help find the area?



Example 2:



- 3.NBT.3 Students apply their understanding of multiplication and the meaning of multiples of 10. For example, 40 is 4 tens and 80 is 8 tens. They can interpret 3 x 20 as 3 groups of 2 tens or 6 groups of ten. They understand that 5 x 60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding they begin to recognize the patterns in multiplying by multiples of 10.

Instructional Resources:

Manipulatives:

- Base Ten Blocks
- Connecting Cubes
- Counters
- Different Size Graph/Grid Paper
- Geoboards
- Hundreds Chart
- Hundreds Chart
- Rubber Bands
- Tiles

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Lesson Seed: #5
- Instructional Strategies: #3, #14, #19
- Extensions for Advanced Learners A: More than Meets the Eye

Literature Connection:

- Sigmund Square Finds His Family by Jennifer Taylor-Cox
- How Tall, How Short, How Far Away? By David A. Adler
- Sir Cumference and the Isle of Immeter by Cindy Meuschwander
- Where We Play Sports: Measuring the Perimeters of Polygons by Greg Roza

Web Links:

- Elementary Mathematics Google Site
  https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/
- PGCPS Universal Design for Learning (UDL)
  http://www1.pgcps.org/UDL/index.aspx?id=127354.
- MSDE Common Core Curriculum Framework
  http://www.mdk12.org/instruction/commoncore/index.html
- Common Core State Standards
  http://www.corestandards.org/
- Place Value Resources
  http://www.aaamath.com/grade3.html#topic3
- Practice Perimeter and Area Exercises
  http://www.mathgoodies.com/lessons/vol1/practice_unit1.html
- Find the Perimeter and Area Playground
  http://www.mathplayground.com/area_perimeter.html
- Perimeter and Area of Polygons
  http://www.mathgoodies.com/lessons/toc_vol1.html
- Place Value, Number Line, Measurement
  http://www.mathsframe.co.uk/resources/category/Partitioning_and_Place_Value.aspx
- Interactive Lessons
  http://learnzillion.com/common_core/math/k-8
- K-5 Math Teaching Resources – Math Games and Hands-on Activities
  http://www.k-5mathteachingresources.com/

Grade 3, Quarter 2 (Suggested Days: 14)
Unit 6: Fractions, Geometry

Common Core Standards:

- **3.NF.1** Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by a parts of size *1/b.*
- **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- **3.NF.2.a.** Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line.
- **3.NF.2.b.** Represent a fraction *a/b* on a number line diagram by marking off *a* lengths*1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
- **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.*

Standards for Mathematical Practice: *Highlighted Standards include the natural way in which students understand and do mathematics; however, other Standards should not be neglected.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Area • Compare • Denominator • Endpoint • Equal Parts • Fraction • Interval • Number Line • Numerator • Partition • Quantity • Unit Fraction • Whole

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- Fractions are numbers.
- Fractions can be used to represent numbers equal to, less than, or greater than 1.
- Fractional parts are relative to the size of the whole or the size of the set.
- There is an infinite number of ways to use fractions to represent a given value.
- A fraction describes the division of a whole (region, set, segment) into equal parts.
- The more fractional parts used to make a whole, the smaller the parts.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- Fractions fall between whole numbers on a number line.

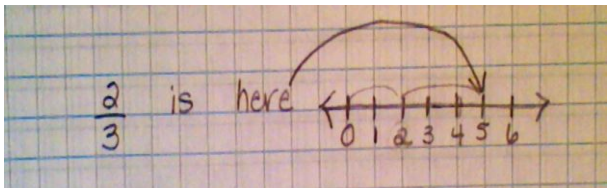
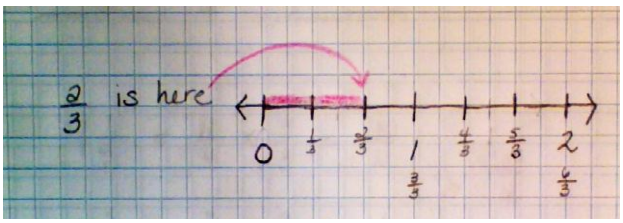
Essential Questions:

A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

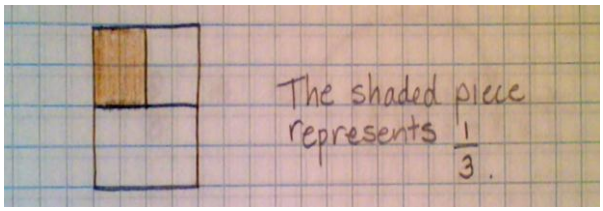
- What is a fraction?
- What are the parts of a fraction?
- How are fractions related to whole numbers?
- Why is the unit fraction an essential concept in understanding fractions in general?
- How can using whole numbers help you better understand fractions of a whole?
- How can I represent fractions in multiple ways?
- How are fractions used in our daily lives?
- How can we represent fractions using visual models, including number lines?
- How does the size of the whole or set impact the relative value of the fraction named?
- How do you express a whole number as a fraction?

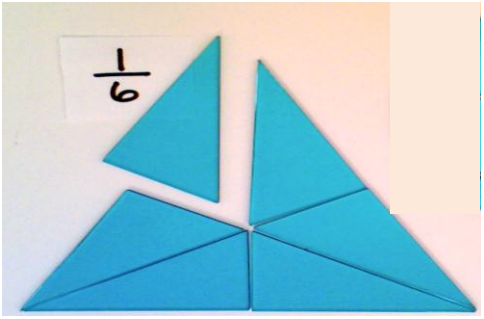
Prior Knowledge:

- This unit formally introduces fractions for the first time in the Common Core. However, fractions have been previously included in grades through geometry and time standards.
- Understand the relationship between the number of equal shares and the size of the shares.
- Knowledge of equal shares of circles and rectangles divided into or partitioned into halves, thirds, and fourths (2.G.3).
- Describe the whole as two halves, three thirds, four fourths.
- Knowledge of circles and rectangles divided into or partitioned into halves, thirds, and fourths.
- Partition circles and rectangles into two, three, or four equal shares: describe the shares using the words: *halves*, *thirds*, *half of*, *a third of*, etc.
- Recognize that equal shares of identical wholes need not have the same shape.
- Knowledge of partitioning rectangles

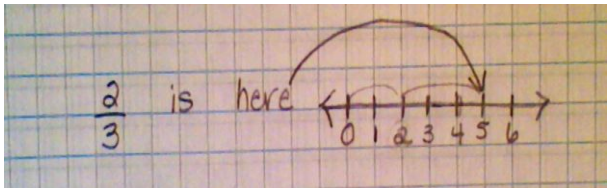
Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students treat numerators and denominators as separate whole numbers. They add both the numerator and the denominator.  	• Provide experiences for students to develop the understanding and come to see a fraction *a*/*b* as *one* number, even though it is written using two whole numbers, *a* and *b*. 

- Students tend to only count pieces in a whole rather than pay attention to whether or not those pieces are *equal* pieces to a whole.

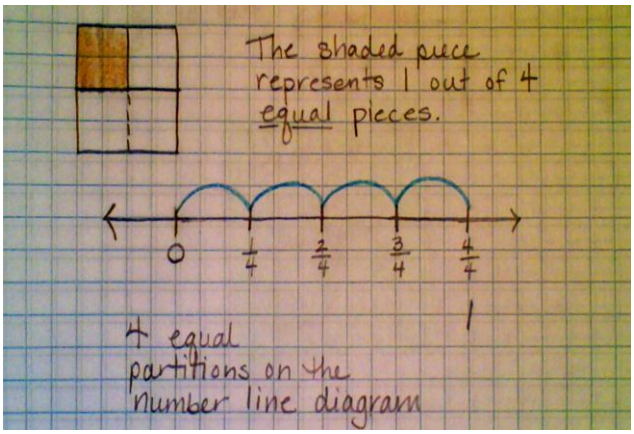


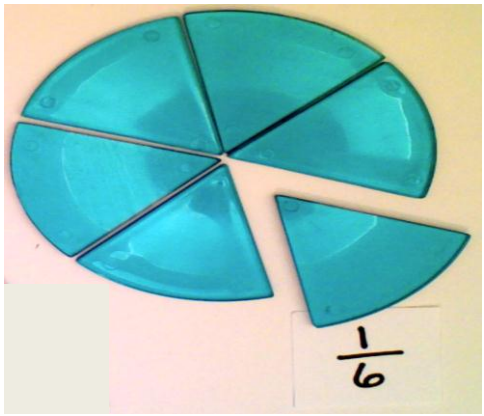


- Students treat numerators and denominators as separate whole numbers. They think the rules of whole number computations also apply to fractions.

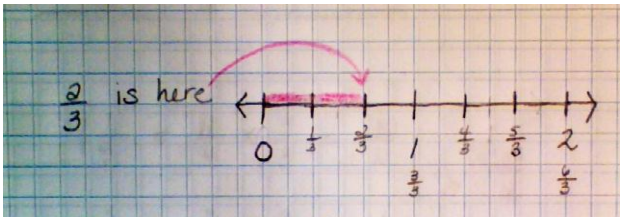


- Provide opportunities for students to experience and understand that equal pieces are necessary when partitioning a shape or number line. Guide students so that they are not just counting parts.





- Provide experiences for students to develop the understanding and come to see a fraction *a/b* as *one* number, even though it is written using two whole numbers, *a* and *b*.



Instructional Notes:

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.
- To develop understanding of fair shares, students should first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.

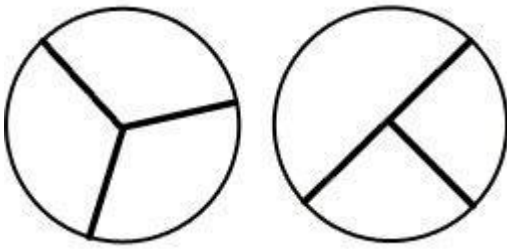
Examples:

1. Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
2. Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?

Some Important Concepts Related to Developing Understanding of Fractions:

- Understand fractional parts must be equal-size

Example **Non-example**



These are thirds **These are NOT thirds**

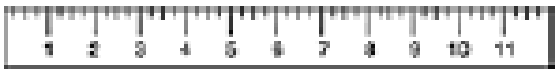
- The number of equal parts tells how many make a whole.
- The size of the fractional part is relative to the whole:
  The number of students in one-half of a classroom is different than the number of students in one-half of a school (the whole in each set is different therefore the half in each set will be different).

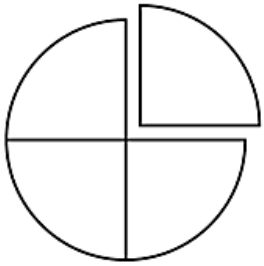
- The denominator names the fraction part that the whole or set is divided into, and therefore is a divisor.
- The numerator counts or tells how many of the fractional parts are being discussed:
  - 3/4 means that there are 3 one-fourths.
  - Students can count *one-fourth, two-fourths, three-fourths.*

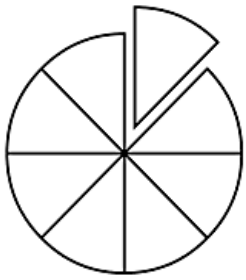
- It is important for students to understand that a fraction is a quantity formed by part of a whole. This understanding is essential in developing number sense with fractions.

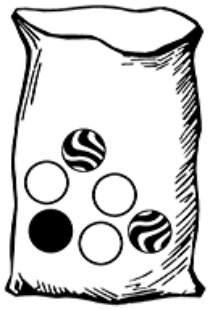
- Students should represent fractional parts in various ways:

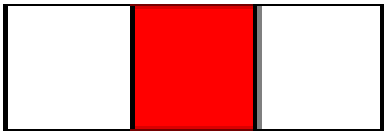
<u>Some Examples of Multiple Representations of Fractions:</u>







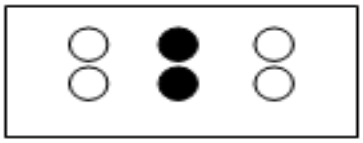




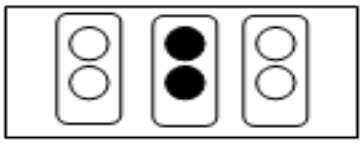
W**hat fraction of the marbles in the bag is striped?**

$$\frac{3}{4}$$

3 - numerator
4 - denominator



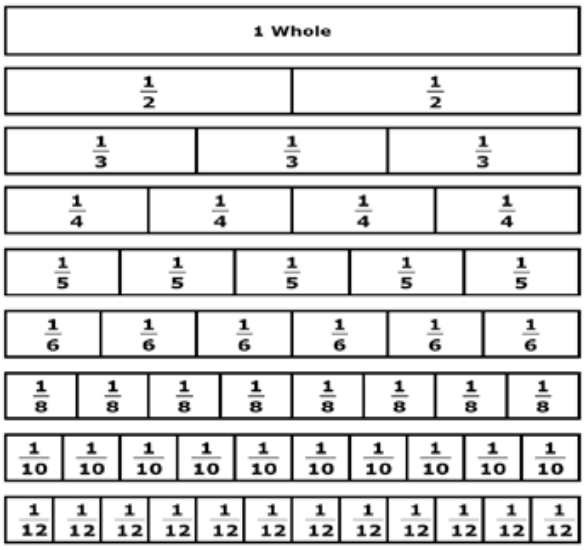
Solution: $\frac{2}{6}$



Solution: $\frac{1}{3}$

What fraction of the set is black?

Fraction Strips

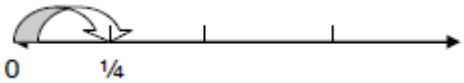


- **3.NF.2.a -2.b** The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that ½ is between 0 and 1).

Example:

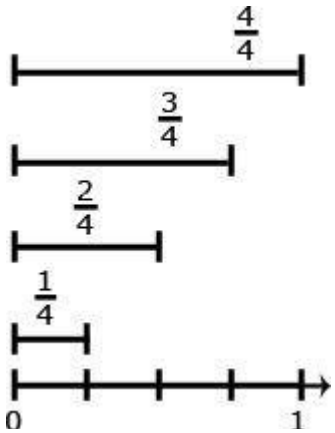
In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (**3.NF.2a**).

Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (**3.NF.2b**).

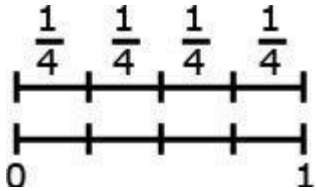


Based on the standard, there are two new concepts students should have time to develop:

1. On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.

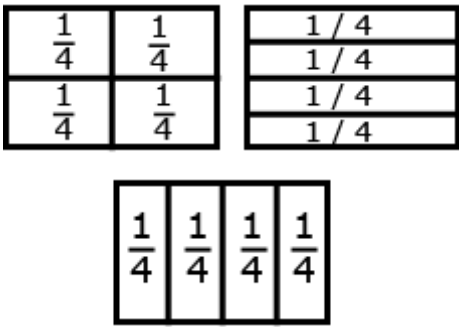


2. Students label each fractional part based on how far it is from zero to the endpoint.



- 3.G.2 Provide opportunities for students to partition shapes into equal parts, recognizing that these parts all have the same area. Students should identify the fractional name of each part and able to partition a shape into parts with equal areas in several different ways.

Example 1:



Example 2:

Area representations of $\frac{1}{4}$

In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.

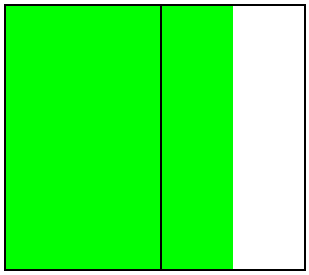
Students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1.

Example: The fraction 3/5 is composed of 3 pieces and each have a size of 1/5.

Students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g., if a whole is partitioned into 4 equal parts then each part is ¼ of the whole, and 4 copies of that part make the whole.

Next, students build fractions from unit fractions, seeing the numerator 3 of ¾ as saying that ¾ is the quantity you get by putting 3 of the ¼'s together.

<u>The Importance of Specifying the Whole</u>



Without specifying the whole it is not reasonable to ask what fraction is represent by the shaded area. If the left rectangle is the whole, the shaded area represents the fraction 3/2; if the entire rectangle is the whole, the shaded area represents ¾.

Instructional Tools/Resources:	
Manipulatives: • Base Ten Blocks • Connecting Cubes • Cuisenaire Rods • Fraction Strips/Circle • Grid/Graph Paper • Number Line • Tiles	**Instructional Supplement** *(Refer to the Elementary Math Google site):* • Lesson Seed: #6, #7 • Instructional Strategies: #12, #43, #53 • Extensions for Advanced Learners C: Fraction Word Riddles • Extensions for Advanced Learners D: World of Fractions Identity Crises • MSDE Unit: Developing Understanding of Fractions as Numbers **Literature Connection:** • My Half Day by Doris Fisher • The Under Achievers by Holly Young • Full House: An Invitation to Fractions by Dayle Ann Dodds • If You Were a Fraction by Speed Shaskan • Whole-y Cow: Fractions Are Fun by Taryn Sounders • Fraction Action by Loreen Leedz • Fractions = Trouble! By Claudia Mills • Piece = Part = Portion by Scott Gifford **Web Links:** • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Mathematics Educator Effectiveness Academy – Unit, Lesson Plans/Seeds http://mdk12.org/instruction/academies/eeaMATH_elementary_ur.html • Fractional Clothesline http://illuminations.nctm.org/LessonDetail.aspx?id=L784 • Irregular partitioned, and un-partitioned areas, lengths, and number lines http://www.conceptuamath.com/strategy1.html • Fractions on a Number Line http://alex.state.al.us/lesson_view.php?id=26348 • Visual Fractions Games http://www.visualfractions.com/Games.htm • Fractional thinking concept map http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_fractions.php • Interactive Lessons http://learnzillion.com/common_core/math/k-8

QUARTER 3

Grade 3, Quarter 3 (Suggested Days:16)
Unit 7: Fractions

Common Core Standards:

- **3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- **3.NF.3.a**. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- **3.NF.3.b.** Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using visual fraction model.
- **3.NF.3.c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.*Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
- **3.NF.3.d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
- **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.

Standards for Mathematical Practice: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Compare • Data • Denominator • Equivalence • Equivalent • Fraction • Line Plot • Number Line • Numerator • Partition • Relational Symbols (>, <, or =) • Horizontal Scale • Visual Fraction Model • Whole

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

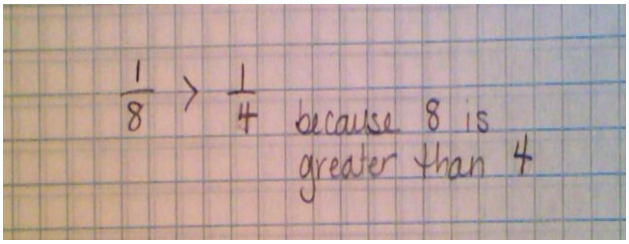
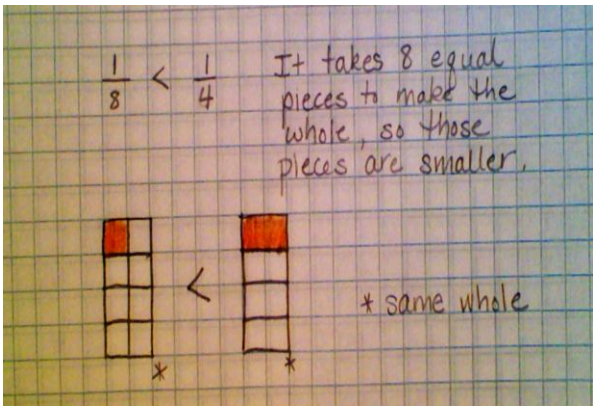
- There is an infinite number of ways to use fractions to represent a given value.
- A fraction describes the division of a whole (region, set, segment) into equal parts.
- The more fractional parts used to make a whole, the smaller the parts.
- An equation is a mathematical statement that uses an equal sign to show that two quantities are equivalent.
- I can compare fractions by comparing the shaded parts of models. < means less than and > means greater than.
- We need to consider the size of the "whole" when comparing fractions of the same size. (Example: 1/2 of a large pie is a different size than ½ of a small pie).
- Larger units can be subdivided into equivalent units (partition).

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How can you compare fractions?
- What are equivalent fractions?
- How are equivalent fractions generated?
- How will your understanding of whole number factors help you understand and communicate equivalent fractions?
- If you have two fractions, how do you know which is greater or has more value?
- Why do we need to consider the size of a whole before comparing the same size fraction? (Think: Comparing ½ of a large pie to ½ of a small pie).
- How can we represent fractions and equivalent fractions using visual models, including number line?
- How do we express a whole number as a fraction" (Example, 5 in the form of 5/1 or locating 6/6 as "1" on a number line.
- Why is unit fraction an essential concept in understanding fractions in general?
- How can you use what you know about whole numbers to help you better understand fractions of a whole?
- What are some ways you can represent fractions?
- How does the size of the whole or set impact the relative value of the fraction named?

Prior Knowledge:

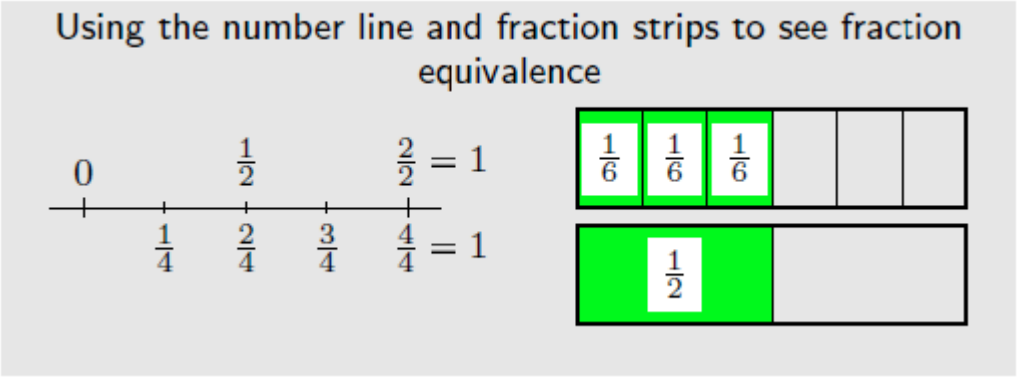
- Fractions have been previously included in grades 1 and 2 through geometry and time.
- Understand the relationship between the number of equal shares and the size of the shares.
- Describe the whole as two halves, three thirds, four fourths.
- Knowledge of circles and rectangles divided into or partitioned into halves, thirds, and fourths.
- Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words: *halves*, *thirds*, *half of*, *a third of*, etc.
- Knowledge of measuring length in whole units using both metric and customary systems.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• The smallest denominator is always the smallest fraction.	• The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.
• The largest denominator is the largest fraction. Students assume this is always true because with whole numbers, they learned that a 6 is larger than a 3.	• Allow students to work with manipulatives to visualize denominators and numerators broken down into their basic parts. Using models such as circles or rectangles, students may easily understand that when the whole have more pieces, the pieces are smaller.
	
• Students don't realize that a numerator of 2 does not mean any two parts, it means two equal parts.	• Provide opportunities for students to use manipulatives to eliminate this misconception.
• Students think all shapes can be divided the same way.	• Present shapes other than circles, squares, or rectangles to prevent students from generalizing that all shapes can be divided the same way. **For example**, have students fold a triangle into eighths. Provide oral directions for folding the triangle: 1. Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex. 2. Fold in this manner two more times. 3. Have students label the fractional parts using fractional notation.

Instructional Notes:

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.
- **3.NF.3 - 3.d** Students should use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should explore equivalent fractions using models, rather than using algorithms or procedures.

Example:



An important concept when comparing fractions is to look at the size of the parts and the number of equal parts.

Example 1:

1/8 is smaller than ½ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

Example 2:

The standard **3.NF.3.c** includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1.

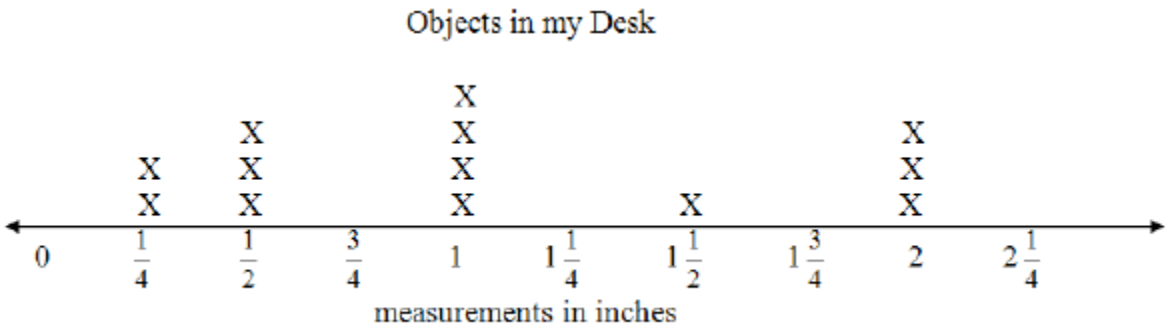
- **3.MD.4** It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler.

  Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.

  Some Important Ideas Related to Measuring with a Ruler:
  - The starting point of where one places a ruler to begin measuring does not have to be at zero.
  - Measuring is approximate. Items that student's measure will not always measure exactly ¼, ½ or one whole inch. Students will need to decide on an appropriate estimate length.
  - Making paper rulers and folding to find the half and quarter marks will help students develop stronger understanding of measuring length.

- Students generate data by measuring and creating a line plot to display their findings.

  Example:



Instructional Tools/Resources:

Manipulatives:

- Base Ten Blocks
- Connecting Cubes
- Cuisenaire Rods
- Fraction Strips
- Grid Paper
- Number Line
- Rulers
- Tiles
- Beaker

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Instructional Strategies: #42, #44, #54 - #55
- Extensions for Advanced Learners C: U.S. Vs.Metric
  Land of the Giant
  Hop Along
  Jump, Jump. Jump
- Extensions for Advanced Learners D: Fraction Action
- MSDE Unit: Developing Understanding of Fractions as Numbers

Literature Connection:

- Full House: An Invitation to Fractions by Dayle Ann Dodds
- Give Me Half! by Stuart J. Murphy
- The WISHING CLUB: A Story About Fractions by Donna Jo Napoli
- Little Numbers And Pictures That Show Just How Little They Are! by Edward Packard
- Go, Fractions! Ages 7-9 (All Aboard Math Reader, Station Stop 3) by Judith Bauer Stamper

Web Links:

- Elementary Mathematics Google Site
  https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/
- PGCPS Universal Design for Learning (UDL)
  http://www1.pgcps.org/UDL/index.aspx?id=127354.
- MSDE Common Core Curriculum Framework
  http://www.mdk12.org/instruction/commoncore/index.html
- Common Core State Standards
  http://www.corestandards.org/
- Mathematics Educator Effectiveness Academy – Unit, Lesson Plans/Seeds
  http://mdk12.org/instruction/academies/eeaMATH_elementary_ur.html
- 3rd Grade Number Activities: Number And Operations - Fractions
  http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html
- Math Resources
  http://www.nylearns.org/module/content/search/advanced.aspx#search
- Fractional Clothesline
  http://illuminations.nctm.org/LessonDetail.aspx?id=L784
- Interactive Glossary
  http://www.ronblond.com/MathGlossary/

	• Math Games and Activities http://illuminations.nctm.org/ActivitySearch.aspx • Lesson Ideas and Videos http://www.teachingchannel.org/ • Tasks and Assessment Tools http://insidemathematics.org/index.php/mathematical-content-standards • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • The Teaching Channel https://www.teachingchannel.org/ • Interactive Lessons to Clarify Standards http://learnzillion.com/common_core/math

Grade 3, Quarter 3 (Suggested Days: 15)
Unit 8: Measurement, Computation

Common Core Standards:

- **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
- **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- **3.NBT.3** Multiply one digit whole numbers by multiples of 10 in the range 10-90(e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Standards for Mathematical Practice: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Addition • AM • Capacity • Elapsed Time • Estimate • Gram • Hour • Inch • Interval • Kilogram • Length • Liter • Milliliters • Minute • Multiple • Mass • Number Line • PM • Second • Standard Unit • Subtraction • Volume

Enduring Understandings (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- The duration of an event can be measured if one knows the start and end times for the event.
- The choice of measurement tools depends on the measurable attribute and the degree of precision desired.
- The weight describes the heaviness an object.
- Some problems can be solved by using objects to act out the actions in the problem. Some problems can be solved by reasoning about the conditions in the problem.
- Capacity is a measure of the amount of liquid a container can hold.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- What strategies can you use to help tell and write time to the nearest minute?
- What connections can you make between a clock and a number line?
- How can you use what you know about number lines to help you figure out how much time has passed between two events?
- What part does elapsed time play in our daily life?
- What types of tools are used to measure volume?
- How can estimating help you to determine liquid volume?
- What does the mass of an object tell you about it?
- What strategies can you use to help you solve problems involving volume and mass?
- How do you differentiate between mass and volume?
- How might measurement errors occur?
- What are tools of measurement? How are they used?
- What is the purpose of standard units of measurements?
- What customary units describe how heavy something is?
- How can you act out and use reasoning to solve problems?
- How can you estimate and measure capacity?

Prior Knowledge:

- Students have experience in telling and writing time from analog and digital clocks to the hour and half hour in Grade 1 and to the nearest five minutes, using a.m. and p.m. in Grade 2.
- Can use the language “quarter to”, “quarter after”, “fifteen minutes after”, and “fifteen minutes before”.
- Understanding of rectangular arrays.
- Counting on, counting by 5’s, 10’s, and 100’s.
- Understand and able to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows.
- Use addition and subtraction to solve one-and two-step word problems.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students often focus on size to determine estimates of mass. They can be confused by a big fluffy object and a tiny dense object.	• Students need opportunities manipulating and handling objects. It is important that teachers do not ask students to estimate the mass of objects until they have had the opportunity to lift the objects and then make an estimate of the mass.

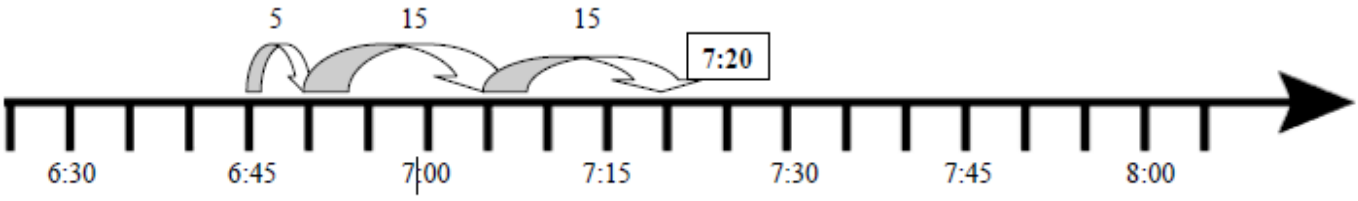
Instructional Notes:

As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.

  Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.
- **3.MD.1** This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve.

  Example: Joseph wakes up at 6:45 a.m. It takes him 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will he be ready for school?



- **3.MD.2** Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram.
- Milliliters may also be used to show amounts that are less than a liter emphasizing the relationship between smaller units to larger units in the same system.
- Students are not expected to do conversions between units, but reason as they estimate, using benchmarks to measure weight and capacity.
- **3.NBT.3 Example**: The problem 40 x 3. Students should think of this as 3 groups of 4 tens or 12 tens. Twelve tens equal 120

  Students should use base ten blocks, diagrams, or hundreds charts to multiply in order to deepen their understanding and to begin to recognize the patterns in multiplying by multiples of 10.

Instructional Tools/Resources:

Manipulatives:

- Analog Clocks
- Beaker
- Different Size Graph/Grid
- Digital Clock
- Geoboards
- Number Line
- Number Line
- Objects to Measure: Volume and Mass
- Rubber Bands
- Scale
- Tiles

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Instructional Strategies: #47

Literature Connection:

- 1001 Bugs to Spot by Emma Helbrough
- The Grapes Of Math by Greg Tang
- How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000? by Helen Nolan
- Lucky Beans by Becky Bertha
- Math Potatoes by Greg Tang
- One Hundred Shoes: A Math Reader (Step-Into-Reading, Step 2) by Charles Ghigna
- Telling Time: How to Tell Time on Digital and Analog Clocks! by Jules Older
- Kids Time With Father Time - Learn How To Tell The Time With Father Time by Daisy Wright
- Addition the Fun Way Book for Kids: A Picture Method of Learning the Addition Facts by Judy Liautaud
- Animal Word Problems Starring Addition and Subtraction (Math Word Problems Solved) by Rebecca Wingard-Nelson
- Domino Addition by Lynette Long
- Count On Clifford by Norman Bridwell
- Animals on Board by Stuart J. Murphy
- The 512 Ants on Sullivan Street by Carol Losi
- My Rows and Piles of Coins by Tololwa M. Mollel

	Web Links: • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Math Games http://www.softschools.com/math/games/mgame2.jsp • K-5 Teaching Resources – Math Games and Hands-on Activities http://www.k-5mathteachingresources.com/ • Math Activities http://illuminations.nctm.org/ActivityDetail.aspx?ID=198 • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • The Teaching Channel https://www.teachingchannel.org/ • Interactive Lessons to Clarify Standards http://learnzillion.com/common_core/math • Math Frames http://www.mathsframe.co.uk/resources/category/3.MD.2._Measure_and_estimate_liquid_volumes_and_masses_of_objects_.aspx

Grade 3, Quarter 3 (Suggested Days: 14)
Unit 9: Properties of Operations, Computation

Common Core Standards:

- **3.OA.5** Apply properties of operations as strategies to multiply and divide.*Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property.)*
- **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ *5 = 8) or properties or operations. By the end of Grade 3, know from memory all products of two one-digit numbers.*
- **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Standards for Mathematical Practice:
These Standards represent behaviors that should be embedded within mathematics instruction.

Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*)
(1) Make sense of problems and persevere in solving them
(6) Attend to precision

Reasoning and Explaining *(Math Talk)*
(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others

Modeling and Using Tools (Math Drawings)
(4) Model with mathematics
(5) Use appropriate tools strategically

Seeing Structure and Generalizing *(Math Structure)*
(7) Look for and make use of structure
(8) Look for and express regularity in repeated reasoning

Mathematical Language:
Terms do not represent an exclusive list and should not be taught in isolation.

- Addend
- Array
- Associative Property
- Commutative Property
- Compare
- Composing
- Counters
- Decomposing
- Difference
- Distributive Property
- Equation
- Factor
- Fluently
- Identity Property of Multiplication
- Mental Computation
- Minuend
- Multiplication
- Multiply
- Order of Operations
- Product
- Properties of Operations
- Quotient
- Reasonableness
- Relationship
- Rounding
- Strategies
- Subtraction
- Subtrahend
- Sum
- Unknown Quantity
- Variable
- Zero Property of Multiplication

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- The order in which the two numbers are added or multiplied does not change the sum or product (Commutative Property).
- You can add or multiply numbers regardless of how they are grouped (Associative Property).
- Multiplying a sum by a number is the same as multiplying each addend by the number and then adding the products (Distributive Property).
- Through the properties of operations (commutative, associative, distributive), we understand the relationships of various mathematical functions.
- The commutative, associative, and distributive properties can be used to develop efficient strategies to multiply.
- Addition, subtraction, multiplication, and division operate under the same properties in algebra as they do in arithmetic.
- Multiplication and division are inverses: they undo each other.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How does drawing an array help us think about different ways to decompose a number?
- How can we determine numbers that are missing on a multiplication table chart by knowing multiplication patterns?
- What strategies can be used to find sums and differences?
- What strategies can be used to solve multiplication or division problems?
- What is the relationship between products and sums, quotients and differences?
- How is the commutative property of multiplication evident in an array model?
- How can an array show the distributive property?
- How can the same array represent both multiplication and division?
- Is there more than one way to multiply or divide a number to get the same product or quotient?
- What do the parts of a division problem represent?
- What happens to the quotient when the divisor increases or decreases?
- How do mathematical operations relate to each other?

Prior Knowledge:

- Composing and decomposing whole numbers.
- Solve one-step problems using multiplication.
- Knowledge of skip counting and explain “why” the pattern works.
- Use addition and subtraction within 100.
- Use addition and subtraction to solve one-and two-step word problems.
- Solve one-step problems using multiplication.
- Explain why addition and subtractions strategies work.

Student Misconceptions and Common Errors:	Addressing the Misconceptions:
• Students think that there is only one way to compose or decompose a number. **For example,** they are unable to see 347 as 34 tens and 7 ones and only see it as 3 hundreds, 4 tens and 7 ones.	• Provide situations for students to decompose numbers in various ways without using the highest place value.
• Students think that division is commutative. They think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations	• Students should use manipulatives to create division situations in order to see that division is not commutative. **Example:** There are 5 candy bars on the table. If there are 25 students in Tony's class, how many candy bars will each student receive?

Instructional Notes:

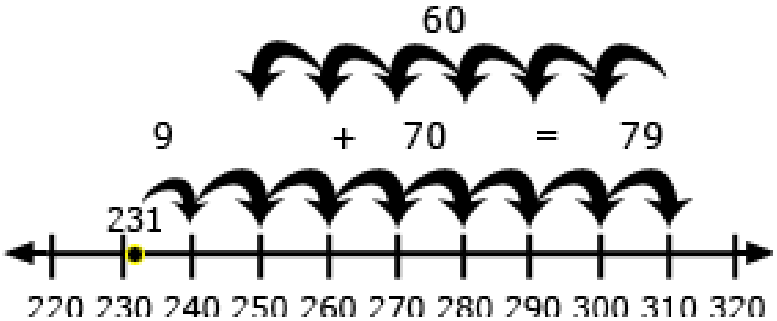
As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.
- **3.OA.5** Student should understand that properties are rules about how numbers work. They should represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

  Provide opportunities for students to use the **distributive property** of multiplication as a strategy for using products they know to solve for products they don't know. **For example**, if students are asked to find the product of 9 x 6, they might decompose 9 into 6 and 3 and then multiply 6 x 6 and 3 x 6 to arrive at 36 + 18 or 54. Students should learn that they can decompose either of the factors**. It is important to note that the students may record their thinking in different ways (this should be encouraged).**

- **3.OA.7** The concept of multiplication can be related to the area of rectangles using arrays. Students should discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column.

- **3.OA.8** **Example 1:** Jose earned 231 points at school last week. This week he earned 79 points. If he uses 60 points to earn free time on a computer, how many points will he have left?



<u>Strategy A:</u> Using the number line as above to describe his or her thinking:

- 231 + 9 = 240 so now I need to add 70 more. 240, 250 (10 more), 260 (20 more), 270, 280, 290, 300, 310
- (70 more). Now I need to count back 60. 310, 300 (back 10), 290 (back 20), 280, 270, 260, 250 (back 60).

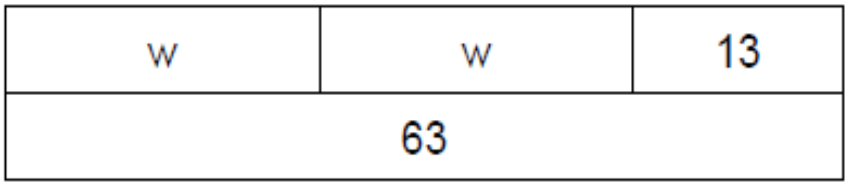
<u>Strategy B:</u> Using Estimation

- A student writes the equation, 231 + 79 – 60 = m and uses rounding (230 + 80 – 60) to estimate.

<u>Strategy C:</u> Using an Equation

- A student writes the equation, 231 + 79 – 60 = m and calculates 79-60 = 19 and then calculates 231 + 19 = m.

Example 2: The football team is going on a trip to the water park. The cost of attending the trip is $63. Included in the price is $13 for lunch and the cost of 2 hats, one red and one green. Write an equation representing the cost of the field trip and determine the price of one hat.



The above diagram helps student write the equation, w + w + 13 = 63. Using the diagram, a student may think, two hats cost $50 ($63-$13) so one hat costs $25.

- **3.OA.9** Students should investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.

Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Instructional Tools/Resources:

Manipulatives:

- Base Ten Blocks
- Connecting Cubes
- Counters
- Hundreds Chart
- Multiplication Table
- Number Line
- Part-Part Whole Mat
- Place Value Math

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Lesson Seed: #8
- Instructional Strategies #31, #36 - #37, #49 - #50
- MSDE Unit 3: Operations and Algebraic Thinking - Solve Problems Involving the Four Operations, and Identify and Explain Patterns in Arithmetic
- MSDE Lesson Seeds: Solve Two-Step Word Problems Involving the Four Operations
  Using Estimation and Mental Math
  Patterns Involving Multiplication
  Number of the Day Activity
  Patterns on the Hundreds Chart

Literature Connection:

- Multiplying Menace: The Revenge of Rumpelstiltskin by Pam Calvert
- Cheetah Math by Ann Whitehead Nagda
- The Doorbell Rang by Pat Hutchins
- Divide and Ride by Stuart J. Murphy
- Anno's Mysterious Multiplying Jar by Masaichire Anno
- Each Orange Had 8 Slices by Paul Giganti
- One Hundred Hungry Ants by Elinor J Pinczes

Web Links:

- Elementary Mathematics Google Site
  https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/
- PGCPS Universal Design for Learning (UDL)
  http://www1.pgcps.org/UDL/index.aspx?id=127354.

	• MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • 3rd Grade Number Activities: Operations And Algebraic Thinking http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html • Bridges in Mathematics http://bridges1.mathlearningcenter.org/files/media/Bridges_Gr3_OnlineSupplement/B3SUP-A2_NumMult_0211.pdf • Interactive Lessons http://learnzillion.com/common_core/math/k-8 • The Teaching Channel https://www.teachingchannel.org/ • Interactive Lessons to Clarify Standards http://learnzillion.com/common_core/math

QUARTER 4

Grade 3, Quarter 4 (Suggested Days: 17)
Unit 10: Fraction, Measurement/Data

Common Core Standards:

- **3.NF.2a**. Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line.
- **3.NF.2b.** Represent a fraction *a/b* on a number line diagram by marking off a length *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
- **3.NF.3.a**. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- **3.NF.3.b.** Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using visual fraction model.
- **3.NF.3.c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
- **3.NF.3.d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model
- **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.

Standards for Mathematical Practice: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Denominator • Endpoint • Equal Parts • Equivalent • Fraction • Interval • Length • Line Plot • Mixed Numbers • Number Line • Numerator • Partition • Region • Relational Symbol (<, >, =) • Relational Symbols • Ruler • Segment • Set • Whole

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- A region can be divided into equal-sized parts in different ways.
- A fraction describes the division of a whole (region, set, segment) into equal parts.
- A fraction is relative to the size of a whole.
- Each fraction can be associated with a unique point on the number line.
- Fractions can be approximated by other fractions that are close.
- Fractions fall between whole numbers on a number line.
- Fractions can be compared by comparing the shaded parts of models. < means less than and > means greater than.
- There is an infinite number of ways to use fractions to represent a given value.
- The more fractional parts used to make a whole, the smaller the parts.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How can you divide a region into equal parts?
- How can a fraction name a part of a group?
- How can different fractions name a part of a whole?
- How can you write fractions in simplest form?
- How can you compare fractions?
- How can you locate and compare fractions and mixed numbers on a number line?
- How can I use what I know about whole numbers to help me better understand fractions of a whole?
- How can I represent fractions in multiple ways?
- How does the size of the whole or set impact the relative value of the fraction named?

Prior Knowledge:

- Knowledge of equal shares of circles and rectangles divided into or partitioned into halves, thirds, and fourths.
- Describe the whole as two - halves, three - thirds, four - fourths.
- Knowledge of equal shares of circles and rectangles divided into or partitioned into halves, thirds, and fourths.
- Partition circles and rectangles into two, three, or four equal shares: describe the shares using the words: *halves*, *thirds*, *half of*, *a third of*, etc.

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• A common misconception is the idea that the smaller the denominator, the smaller the piece or a part of a set, or the larger the denominator, the larger the piece or a part of a set. This misconception is based comparing whole numbers. Students learned that 6 is greater than 3 and they assume that this is true when looking at denominators., For Example, 1/6 is greater than 1/3.	• Use models that include circles, squares, and rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. Students may easily understand that in order for the whole to have more pieces, the pieces must be smaller. The use of different models, such as fraction bars and number lines, allows students to compare unit fractions and reason about their sizes.

Instructional Notes:

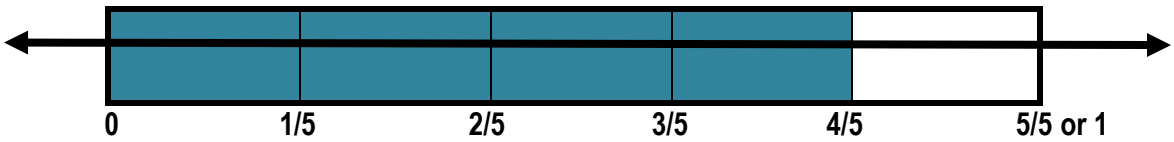
As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.

- **3.NF.2a-2b**

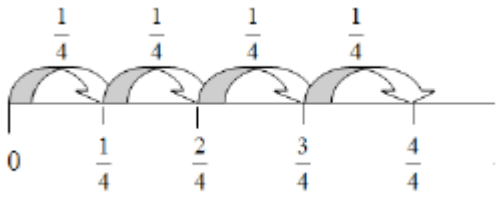
Example 1: To plot 4/5 on a number line, there are 5 equal parts with 4 of the 5 equal parts.

5 equal parts make the whole

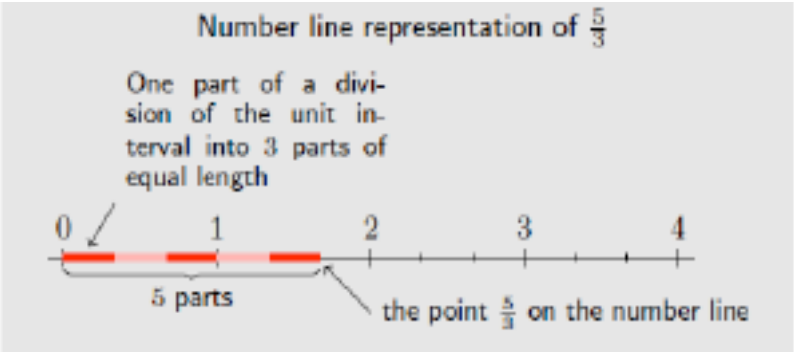


4 of the 5 equal parts shaded represent the fractional amount 4/5

Example 2: In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (**3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (**3.NF.2b**).

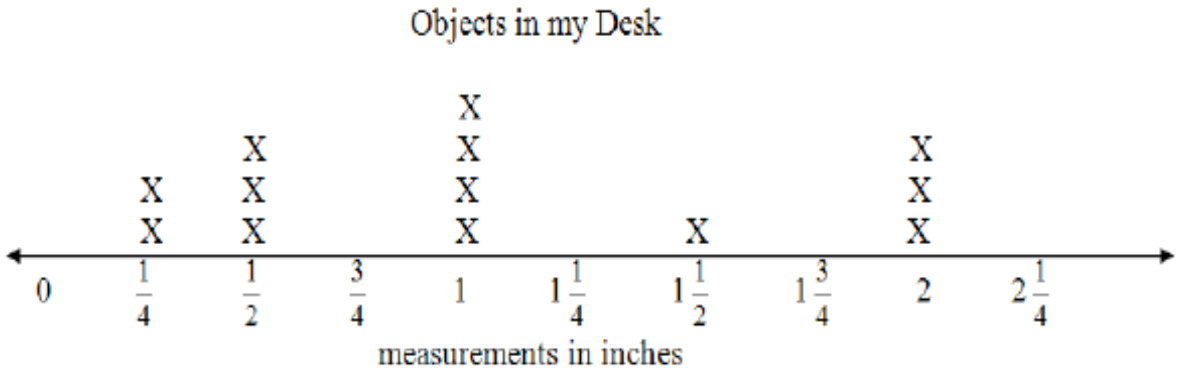


Example 3:



- **3.MD.4** Students generate data by measuring and creating a line plot to display their findings.

Example:



Instructional Tools/Resources:	
Manipulatives: • Base Ten Blocks • Connecting Cubes • Cuisenaire Rods • Fraction Strips • Grid/Graph Paper • Number Line • Ruler • Tiles	**Instructional Supplement** *(Refer to the Elementary Math Google site):* • Instructional Strategies: #45 • MSDE Unit: Develop Understanding of Fractions as Numbers **Literature Connection:** • My Half Day by Doris Fisher • The Under Achievers by Holly Young • Full House: An Invitation to Fractions by Dayle Ann Dodds • If You Were a Fraction by Speed Shaskan • Whole-y Cow: Fractions Are Fun by Taryn Souders • Fraction Action by Loreen Leedz • Fractions = Trouble! by Claudia Mills • Piece = Part = Portion by Scott Gifford **Web Links:** • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Mathematics Educator Effectiveness Academy – Unit, Lesson Plans/Seeds http://mdk12.org/instruction/academies/eeaMATH_elementary_ur.html • Fractional Clothesline http://illuminations.nctm.org/LessonDetail.aspx?id=L784 • Irregular partitioned, and un-partitioned areas, lengths, and number lines http://www.conceptuamath.com/strategy1.html • Visual Fraction Games http://www.visualfractions.com/Games.htm • Fraction on a Number Line http://alex.state.al.us/lesson_view.php?id=26348 • Fractional Thinking Concept Map http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_fractions.php

Grade 3, Quarter 4 (Suggested Days: 15)
Unit 11: Geometry, Measurement/Data

Common Core Standards:

- **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.*
- **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
- **3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
- **3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Standards for Mathematical Practice:
These Standards represent behaviors that should be embedded within mathematics instruction.

Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*)
(1) Make sense of problems and persevere in solving them
(6) Attend to precision

Reasoning and Explaining *(Math Talk)*
(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others

Modeling and Using Tools (Math Drawings)
(4) Model with mathematics
(5) Use appropriate tools strategically

Seeing Structure and Generalizing *(Math Structure)*
(7) Look for and make use of structure
(8) Look for and express regularity in repeated reasoning

Mathematical Language:
Terms do not represent an exclusive list and should not be taught in isolation.

- Area
- Bar Graph
- Capacity
- Categories
- Centimeter
- Data
- Gram
- Hexagon
- Interval
- Kilogram
- Length
- Liter
- Mass
- Millimeter
- Number Line
- Parallel
- Parallelogram
- Perimeter
- Picture Graph
- Property
- Quadrilateral
- Rectangles
- Rhombus
- Right Angles
- Scale
- Scaled Bar Graph
- Scaled Picture Graph
- Square
- Standard Unit
- Survey
- Table
- Trapezoid
- Volume

Enduring Understandings (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- Two-dimensional shapes can be described using properties that are shared between the shapes.
- Shapes can be classified.
- Quadrilaterals can be classified according to the relationship of their sides to each other.
- Quadrilaterals have specific characteristics of the angles and the relationship between opposite sides.
- Data displays describe and represent data in alternative ways.
- Attributes of shapes, including angles and equal faces, help you to understand objects and compose new shapes.
- Graphs convey data in a concise way.
- Each type of graph is most appropriate for certain kinds of data.
- The key for a picture graph determines the number of pictures needed to represent each number in the set.
- In a bar graph, the scale determines how long the bar needs to be to represent each number in a set of data.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- What are some special names for quadrilaterals?
- How can you use the attributes of two-and three-dimensional shapes to classify them?
- What is a two-dimensional shape?
- What is a quadrilateral?
- How can angle and side measures help us to create and classify quadrilaterals?
- How can you partition shapes into equal areas?
- How do I decide what increments (intervals) to use for my scale?
- What strategies will help me solve for an unknown side when finding perimeter?
- How can you estimate and measure length?
- How can you estimate and measure capacity?
- What customary units describe how heavy something is?
- How do you differentiate between mass, weight, and capacity?
- Why are units important in measurement?
- How can surveys be used to collect data and answer questions?
- How can graphs be used to display data gathered from a survey?
- When is it appropriate to use a line plot?
- How are tables, bar graphs, and line plot useful ways to display data?
- How do you determine how much a symbol in a pictograph represents?
- Why does "what" we measure influence "how" we measure?
- Why display data in different ways?
- How are a bar graph and picture graphs related? What are their differences?
- How does the type of data influence the choice of display?
- How do you determine what a symbol in a picture graph represents?
- How can graphs be used to compare related data?

Prior Knowledge:
• In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. • Experiences with informal reasoning about particular shapes through sorting and classifying using geometric attributes. • Build and draw shapes given the number of faces, number of angles and number of sides. • Partition circles and rectangles into two, three, or four equal shares: describe the shares using the words: *halves*, *thirds*, *half of*, *a third of*, etc. and describe the whole as two halves, three thirds or four fourths. • Tell and write time in hours, half-hours and to the nearest five minutes and using a.m. and p.m. • Understand that larger units can be subdivided into equivalent units (partition). • Understand that the same unit can be repeated to determine the measure (iteration).

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students may describe a square as nonrectangular. They do not recognize that a square is a rectangle.	• Have students use straws to make figures and change the angles to see the relationships between a rectangle and a square. The square has all of the properties of a rectangle. This strategy will help students to see the interrelationships between the shapes and clarify the misconception.
• Students do not see or understand the actual relationship between the hour and the minute hand.	• Avoid the use of paper plate clocks. Use analog clocks, digital clocks, or a number line to represent time intervals in minutes. This is not adequately represented on student-made clocks.
• Students think it is not important to label the measurement or choose the correct unit of measurement.	• Provide multiple experiences where students are estimating and measuring and recording finding using the correct unit of measurement.
• Although intervals on a bar graph are not in single units, students count each square as one.	• To address this misconception, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip-counting when determining the value of a bar since the scale is not in single units

Instructional Notes:

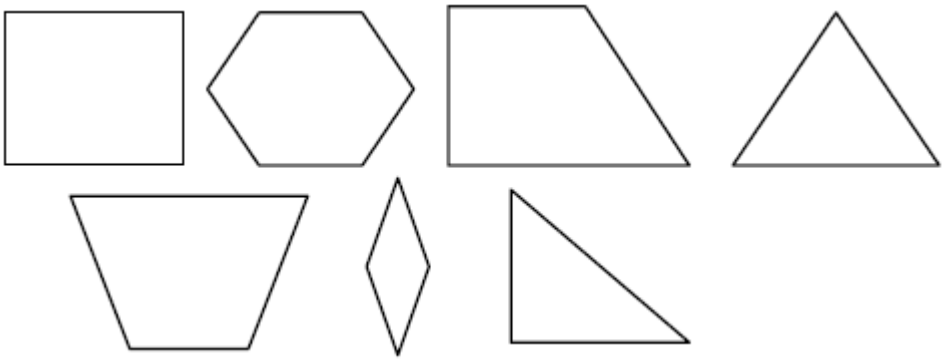
As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
- Repetition of vocabulary in instruction and explanation encourages the students to follow the model using specific language.
- When introducing new manipulatives, allow a few minutes of exploration time.
- Be sure to incorporate the Enduring Understandings and the Essential Questions as a foundation for your instruction.
- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
- Review the Progression for Grades K-5 to see the development of the understanding of the Common Core Standards, which is also the guiding information for the PARCC Assessment development. Access document from the Elementary Mathematics Google site.

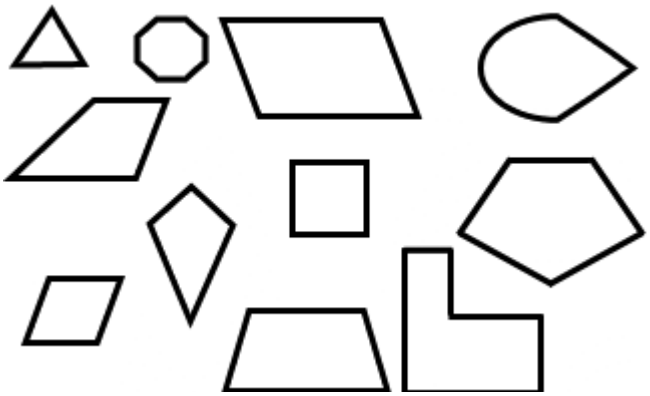
- **3.G.1** Students are expected to identify and describe properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification.

  Students should conceptualize that a quadrilateral must be a closed figure with four straight sides and should notice characteristics of the angles and the relationship between opposite sides.

  Example 1: Students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes.



Additional Shapes for Classification:



Students should classify shapes by attributes and draw shapes that fit specific categories.

Example: Draw a picture of a quadrilateral.
Draw a picture of a rhombus. How are they alike? How are they different?
Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

Instructional Tools/Resources:

Manipulatives:

- Analog Clocks
- Balance Scales
- Beakers
- Digital Clocks
- Graduated Cylinders
- Grid Paper
- Number Line
- Objects to Weigh
- Ruler
- Weights: Grams and Kilograms

Instructional Supplement *(Refer to the Elementary Math Google site):*

- Lesson Seed: #9, #10
- Instructional Strategies: #13, #28, #30
- Extensions for Advanced Learners A: Picture This!
  I'm Seeing in 3-D!
  Tessellation Quilt

Literature Connection:

- Grandfather Tang's Story by Ann Tompert
- Rhombus by Sheila Rivera
- Sea Shapes by Suse MacDonald
- The Shape Of Things by Dayle Ann Dodds
- So Many Circles, So Many Squares by Tana Hoban
- Icky Bug Shapes by Jerry Pallotta
- When a Line Bends . . . A Shape Begins by Rhonda Gowler Greene
- Pastry School in Paris: An Adventure in Capacity by Cindy Neuschwander
- The Great Pyramid of Giza: Measuring Length, Area, Volume, and Angles by Janey Levy

	• Millions to Measure by David M. Schwartz **Web Links:** • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Practice Perimeter and Area Exercises http://www.mathgoodies.com/lessons/vol1/practice_unit1.html • Partitioning and Place Value Resources http://www.mathsframe.co.uk/resources/category/Partitioning_and_Place_Value.aspx • Junior Architects – Finding Perimeter and Area http://illuminations.nctm.org/LessonDetail.aspx?ID=L651 • Find the Perimeter and Area Playground http://www.mathplayground.com/area_perimeter.html

Grade 3, Quarter 4 (Suggested Days: 15)
Unit 12: Measurement

Common Core Standards:

- **3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement.
- **3.MD.5.a.** A square with side length 1 unit, called a "unit square," is said to have "one square unit" of area, and can be used to measure area.
- **3.MD.5.b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
- **3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- **3.MD.7** Relate area to the operations of multiplication and addition.
- **3.MD.7.c**. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.
- **3.MD.7.d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Standards for Mathematical Practices: *These Standards represent behaviors that should be embedded within mathematics instruction.*	**Mathematical Language:** *Terms do not represent an exclusive list and should not be taught in isolation.*
Habits of Mind of a Productive Mathematical Thinker (*Math Sense Making*) (1) Make sense of problems and persevere in solving them (6) Attend to precision **Reasoning and Explaining *(Math Talk)*** (2) Reason abstractly and quantitatively (3) Construct viable arguments and critique the reasoning of others ***Modeling and Using Tools (Math Drawings)*** (4) Model with mathematics (5) Use appropriate tools strategically **Seeing Structure and Generalizing *(Math Structure)*** (7) Look for and make use of structure (8) Look for and express regularity in repeated reasoning	• Additive • Area • Area Measurement • Decomposing • Distributive Property • Gap • Improvised • Nonstandard Units • Overlap • Plane Figure • Rectilinear • Side length • Square centimeter • Square feet • Square inch • Square meter • Square Unit • Tiling

Enduring Understanding (Big Ideas):
Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.

- Area is an attribute used to describe and measure two-dimensional figures.
- Area covers a certain amount of space using square units.
- Measurement processes are used in everyday life to describe and quantify the world.
- The region inside a shape is its area and can be measured using square units.
- Interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows.

Essential Questions:
A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.

- How are area and perimeter different?
- How do we choose the appropriate unit of measurement?
- How can you apply rules of the distributive property to find the area of a rectilinear figure?

Prior Knowledge:

- Solve one-step problems using multiplication.
- Knowledge of multiplication and division
- Knowledge of repeated addition

Common Student Misconceptions and Errors:	Addressing the Misconceptions:
• Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is the total distance around a figure, which is perimeter.	• Pose problems situations that require students to explain whether they are finding the perimeter or area. Also use tiling to show the area is the amount of square units inside a figure.

Instructional Notes:

As standards repeat during this unit, use pre-assessment data to identify areas of weakness and re-teach through the integration of standards. Provide further in depth conceptual activities, more high level tasks to promote deep understanding, opportunities for students to apply concepts using real world experiences (problems in context), and performance-based tasks to support students in achieving mastery of standards.

- Information, resources, and instructional strategies for **ALL** learners including students with disabilities and English Language Learners (ELL) students can be found on the **PGCPS Universal Design for Learning (UDL)** webpage.
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- It is vital that the students use concrete or virtual manipulatives to represent the numbers with which they are working.
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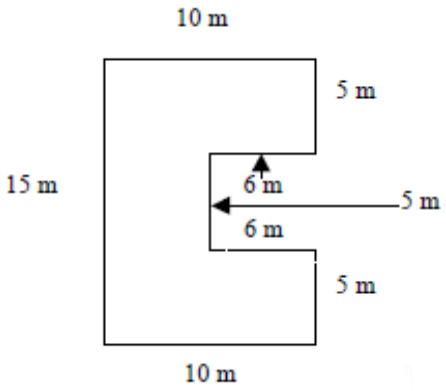
- **3.MD.7.c** Provide experiences for students to tile rectangles then multiply the side lengths to show it is the same.

Example:

1	2	3	4
5	6	7	8
9	10	11	12

To find the area one could count the squares or multiply 3 x 4 = 12.

- **3.MD.7.d Example:** The figure below represents Mrs. Thomas's vegetable garden. What is the total area? How could the figure be decomposed to help find the area?



Instructional Tools/Resources:	
Manipulatives: • Counters • Different Size Graph/Grid Paper • Geoboard • Rubber Bands • String • Tiles	**Instructional Supplement** *(Refer to the Elementary Math Google site):* • Extensions for Advanced Learners A: Rover's All Over Tile It **Literature Connection:** • You Can Count on Monsters: The First 100 Numbers and Their Characters by Richard Evan Schwartz • Minnie's Diner: A Multiplying Menu by Dayle Ann Dodds • Spunky Monkeys on Parade by Stuart J. Murphy • Famous Bridges Of The World: Measuring Length, Weight, And Volume by Yolonda Maxwell • Building Washington, D.C: Measuring the Area of Rectangular Spaces by Barbara M. Linde **Web Links:** • Elementary Mathematics Google Site https://sites.google.com/a/pgcps.org/elementary-mathematics-russ/ • PGCPS Universal Design for Learning (UDL) http://www1.pgcps.org/UDL/index.aspx?id=127354. • MSDE Common Core Curriculum Framework http://www.mdk12.org/instruction/commoncore/index.html • Common Core State Standards http://www.corestandards.org/ • Kids Math: Area http://ethemes.missouri.edu/themes/682?locale=en • Math: Geometry http://ethemes.missouri.edu/themes/232 • Math Resources http://www.commoncoreconversation.com/math-resources.html#sthash.oLDG5vLS.dpbs • Math Resources http://www.nylearns.org/module/content/search/advanced.aspx#search

Appendix

Planning for Instruction

5Es Mathematics K-12 Lesson Planner

The 5 E's Model for the teaching of mathematics is based on the constructivist approach to learning. Learning new concepts or attempting to understand something familiar in greater depth, is not a linear process. **Therefore, suggested time slots for each of the stages within the model may vary according to the activity planned for the lesson design.**

Lesson Title: **Date:** **Standard:** **Materials Needed** **(1e: Materials and Resources) :** **Data Points:**	**5 E's**	**Questions for Planning/ Lesson Planning Notes** **Please note that FFT Components for Domain I are referenced in the questions for planning by Indicator; for example** **Value, Sequence and Alignment-See Lesson Readiness; Concept Development** **Balance: See Lesson Readiness; Concept Development** **Clarity- See Objective** **Suitability for Diverse Leaners- See Lesson Readiness; Concept Development; Learning Activity; Flexibility and Fluidity** **As teachers plan for instruction, please use the guiding questions for lesson planning as appropriate.**
Time Frame		
Engagement	**Engagement (Individual, Small Group or Whole Group Work)** ❑ Objective stated written/orally ❑ Pre-Assessment ❑ Connecting to Prior Knowledge ❑ Learning Activity Set-Up ❑ Multiple Entry Points ❑ Homework review	**Lesson readiness:** ❑ What data indicates that students are ready for this lesson? **(1c:Balance; Suitability for Diverse Learners)** ❑ Where does this lesson fall in the sequence of learning? **(1c: Value, Sequence, and Alignment)** ❑ How are the concept outcomes connected to previous and future learning? **(1c: Value, Sequence, and Alignment)** ❑ How does this lesson align to the progression of this unit? **(1e: Lesson and Unit Structure)** ❑ How can I connect students' enduring understanding of concept to other concepts/concepts to other disciplines? What are the essential questions?**(1c Value sequence and alignment)** **NOTES:** **Objective (1c:Clarity)** **Is the objective:** ❑ Measurable (What will the students learn?) ❑ Written with verbs for expectations of high levels of rigor ❑ Written as an outcome-not an activity ❑ Specific, doable, assessable in the allotted time (How will I assess student knowledge?) **NOTES:** **Concept Development:** ❑ How can I set up learning activities to aid students in learning the new mathematics in multiple

		ways with multiple representations? **(1c: Balance)** ❑ What questions do I ask if students have difficulty getting started? What questions do I ask to advance thinking? **(1c: Suitability for Diverse Learners)** ❑ How will this experience help students develop proficiency with one or more of the course standards? **(1c Value sequence and alignment)** **NOTES:**
Exploration	***Exploration (Various Groupings)*** ❑ Think/ Wait Time ❑ Hands on Lab ❑ Manipulatives & Tools ❑ Modeling Questions ❑ Co-operative learning (i.e., Jigsaw, Think- Pair-Share, Flexible Grouping) ❑ Use of Technology	**Learning Activity:** **How does the clear and sequenced activity:** ❑ Engage students and advance them through the content? **(1e: Lesson and Unit Structure)** ❑ Support instructional outcomes and reflect important concepts? **(1e: Lesson and Unit Structure)** ❑ Provide a variety of appropriately challenging materials and resources to advance student learning of the concept understanding and meet the differentiated needs of students in the class? (**1e: Instructional Materials/Resources; 1c Suitability for Diverse Learners)** ❑ Provide assessing and advancing questions that will develop understanding of the concept? **(1e: Lesson and Unit Structure)** ❑ Provide models and/or tools that will help students advance their understanding of the concept? (How will misconceptions be addressed?) **(1e: Lesson and Unit Structure)** ❑ Help students develop proficiency with one or more of the Mathematical Practices? **NOTES:** **Student Groups:** **How will student groups: (1e: Instructional Groups)** ❑ Be formed to intentionally support and advance student learning? ❑ Be expected to work together? ❑ Use appropriate technology? ❑ Be scaffolded to help students develop proficiency with one or more of the course standards? **(1c Value sequence and alignment)** **NOTES:**
Explanation	**Explanation *(Whole Class/Student Response)*** ❑ Concept Web ❑ Mind Map	**Concept Explanations:** ❑ Engage students to clarify misconceptions and errors. What questions can I provide to aid students in correcting misconceptions? **(1e: Lesson and Unit Structure)**

	❑ Partner share ❑ Vocabulary development ❑ Student centered ❑ Evaluation ❑ Portfolio	❑ Encourage students to explain their observations and findings in their own words. **(1c:Clarity)** ❑ Facilitate clarification of new vocabulary. What strategies and tools will I use to help students actively formulate new vocabulary? **(1e: Instructional materials and resources)** **NOTES:** **What open ended questions will I ask to:** ❑ Promote higher order thinking and to assess student learning of the concept? ❑ Advance student learning? ❑ Clear misconceptions? **NOTES:**
Elaboration	**Elaboration *(Develop Flexibility and Fluidity: Explicit Teaching/Guided Practice)*** ❑ KWL Chart ❑ Intervention Activities ❑ Problem of the Week ❑ Modeling Concepts ❑ Problem of the Week ❑ Graphic Organizers ❑ Games ❑ Technology	**Flexibility and Fluidity:** ❑ What new experiences will I provide for students to expand their understanding and connect to real-world situations and other disciplines? **(1c: Suitability for Diverse Learners)** ❑ What scaffolding and modeling will I use to aid students in extending and explaining concepts being explored? **(1e: Instructional Materials and Resources)** ❑ What questions will I ask students to encourage them to apply concepts and skills to new situations? **(1e: Learning Activities; Instructional Materials; Lesson and Unit Structure)** ❑ How will I aid students in linking mathematical vocabulary to present and future concepts? **(1e: Learning Activities)** **NOTES:**

Evaluation	Evaluation	Evaluation (1e:Lesson and Unit Structure)
	❑ Gallery Walks ❑ Journal Entries ❑ Exit Slips ❑ Quick writes ❑ Student interviews ❑ Buddy Check ❑ White Boards ❑ Answer Cards	❑ How will evidence be collected to determine that the students have attained the learning objectives? ❑ How will evidence be used to inform instructional decisions? ❑ How will I examine students' work and identify needs to plan future instruction? ❑ How and when will I provide extensive feedback to address growth towards understanding**?** ❑ What is my timeframe for providing feedback to students with useful information to adjust their current learning approaches and take ownership of their learning? **NOTES:**

ASSISTING ENGLISH LANGUAGE LEARNERS

To assist English Language Learners' participation in the class discussion:
Allow time for students to first work individually, then talk in small groups (pairs) and, finally, have the group's report to the whole class and take part in a whole-group discussion of the mathematical ideas. Consider think-pair-square-share, where pairs share their answers with another pair ("square") before the whole-class math discussion reconvenes.
Reinforce appropriate language as students communicate their ideas (e.g., revoice a student's contribution in mathematically and grammatically correct language). Ask students if you have captured what they said.
Develop a word wall and insert new terminology as it is introduced. Consider including a visual representation of mathematical terms, as appropriate. (For example, one side of a word wall card shows the word and the other side has a pictorial representation.
Students can use the word wall interactively to check their understanding. Continually refer to it until the words become a part of student dialogue, and then transfer the terms to the class "archives" for reference as needed.)
Expect students to incorporate appropriate mathematical vocabulary in their discussions after terms have been introduced.

Examples of Sensory, Graphic and Interactive Support for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
• Real-life objects (realia) • Manipulatives • Pictures and photographs • Illustrations, diagrams and drawings • Magazines and newspapers • Physical activities • Videos and Films • Broadcasts • Models and figures	• Charts • Graphic Organizers • Tables • Graphs • Timelines • Number Lines	• In pairs or partners • In triads or small groups • In a whole group • Using cooperative group structures • With the Internet (Web sites) or software programs • In the native language (LI) • With mentors

Strategies for Adapting Mathematical Instruction to Reach All Learners

Strategies to Aid in Levels of Learning Mastery in Mathematics

Students should move through six levels of mastery to truly learn and retain mathematical concepts. They are as follows:

- Level One: Connects new knowledge to existing knowledge and experience
- Level Two: Searches for concrete materials to construct a model or show a demonstration of the concept
- Level Three: Illustrates the concept by drawing a diagram to connect the concrete example to a symbolic picture or representation
- Level Four: Translates the concept into mathematical notation using number symbols, operational signs, formulas, and equations
- Level Five: Applies the concept correctly to real-world situations, projects, and story problems
- Level Six: Can teach the concept successfully to others or can communicate it on a test

Difficulty with Understanding Math Concepts Without Concrete Examples

- Work the problems with the student explaining how to associate concrete examples with each problem (e.g., two-color counters to represent addition and subtraction of integers).
- Have student use sets of objects to practice addition, subtraction, multiplication, and division problems.
- Model transferring from the concrete level (manipulatives) to the representational (drawing pictures) to the abstract level (symbols and numbers) in solving problems.
- Vary group size (flexible grouping) to allow progress through skills at a comfortable rate.
- Introduce new concept through everyday situations as opposed to worksheets.
- Have a peer tutor assist the student in solving math problems.
- Make the connection between math concept and real-life situations.
- Teach the student concepts one at a time before pairing the concepts (e.g., dimensionality, size, space, shape).
- Begin with a simple version of the task you intend to pose.

Difficulty Changing from One Math Operation to Another

- Use visual cues (stop signs or red dots) on the paper when the student must change operations.
- Use color coding (addition signs, green; subtraction signs, red). Gradually reduce the use of colors as the student demonstrates success.
- Have the student practice recognizing a series of math symbols (e.g., +,-, ÷, x).
- Make the math operation symbols, next to the problems, extra large in order that the student will be more likely to observe symbols.
- Have student highlight operations of each problem before beginning to solve math problems.
- Initially use a written reminder beside each problem to indicate which operation is to be used, then gradually reduce the use of reminder.
- Reduce the amount of information on a page to minimize visual distractions for the student.
- Have student talk through math problem while solving, in order to identify errors he/she is making.

Strategies for Adapting Mathematical Instruction to Reach All Learners
Difficulty in Working Math Problems from Right to Left
• Use colored arrows to indicate where the student begins to work problems (e.g., right to left). • Provide a demonstration on paper of the first problem as an example. • Put the math problem on graph paper or vertically lined paper to emphasize columns, with directions to begin each problem at the right. • Make sure the student has mastered place value concepts and understands that columns to the left have higher values than those to the right. • Require the student to solve math problems using a place value template. • Have student use a calculator to solve math problems. • Provide practice in math by using computer software programs that provide immediate feedback.
Difficulty with Remembering Math Facts
• Have student use calculators to reinforce learning of the math facts. • Provide student with many concrete experiences to help learn and remember math facts. • Use a computer software program (e.g., FASTT Math) that provides immediate feedback to practice. • Provide opportunities for student to apply facts to real-life situations. • Develop a math facts reference sheet for the student to use at his/her desk when solving math problems. • Gradually increase the number of facts for student to remember.
Difficulty in Following Steps in Math Problems
• Use graphic organizers to indicate which step is to be done. Gradually reduce cues. • Color-code math steps next to math problems. • Provide an example of the first problem, with steps on the paper as an example. • Have steps in solving math problems readily available on graphic organizer, chalkboard, bulletin board, on student's desk, etc. • Have the student check answers to math problems on a calculator. • Have student equate math problems to real-life situations in order that he/she will better understand the steps involved in solving the problem. • Have student verbalize the problem solving steps to self or teacher.
Difficulty with Solving Math Word Problems
• Have student orally analyze the steps (e.g., "What is given?", "What is asked?", "What operation is used?"). • Prior to introducing complex word problems, present the student with phrases to be translated into numbers (e.g., *n* divided by 4: n/4). • Use word problems that are related to the experiences of the student.

Strategies for Adapting Mathematical Instruction to Reach All Learners
• Have student solve word problems by manipulating objects and stating the process used. • Model problem solving and make problem solving the reason for computation. • Have student restate math word problems in his/her own words. • Provide student with a graphic organizer for problem solving (e.g., SOLVE, STAR). • Have student check his/her word problems using a calculator. • Break down multi-step problems into smaller parts and solve. • Have student underline, highlight or circle key math words or directions. • Discuss questions in multiple-choice answers and reasons why choices will not work. • Present problems with answers but without signs and have students supply the correct sign.
Difficulty with Math Problems Involving Fractions or Decimals
• Have students solve math problems involving fractions and decimals using tangible objects (fraction circles, fraction strips, decimal squares, or 100th place value block, grid paper with 10x10 centimeter blocks, etc.). • Cut pieces of paper into equal numbers or use fraction strips (e.g., fourths, sixths, tenths); have the student add fractions together, subtract fractions, compare fractions, etc. • Ask the student to do skip counting on a number line or ruler by fraction. • Have the student do skip counting by decimals (e.g., .2, .4, .6, .8). • Develop a reference sheet of fractions and decimals for the student to keep at his/her desk. • Provide practice in fractions and decimals by using computer software program that gives immediate feedback. • Provide the student with concrete examples and opportunities to apply these concepts in real-life situations (sharing a pizza, money concepts, measuring cloth, following a recipe, etc.).
Difficulty in Understanding Algebraic Expressions
• Use manipulative objects (e.g., Algebra blocks) to help the student with problem representations and to provide a visual image moving from the concrete to the pictorial to the symbolic expression (abstract). • Connect or pair expressions that do not have variables to expressions with variables. • Use visual organizers, such as structures worksheets, prompt cards, or graphic organizers to aid students to recall problem solving steps in word problems. • Review the meaning of the four basic operations in terms of actions in problems: 1. Addition is the joining together of two or more sets of objects. 2. Subtraction may be the removal of objects from a set or the finding of a missing part of a set. 3. Multiplication is the combining of several equal-sized sets of objects.

Strategies for Adapting Mathematical Instruction to Reach All Learners
4. Division is the separation of a given amount of objects into equal parts. • Give student several story situations that involve one or more operations and have him/her demonstrate actions with cubes and counters (cubes represent the unknown, counters represent individual objects). • After student uses cubes and counters to build expressions, have him/her draw pictures of objects (square for cube and circle for counter) and write symbolic expressions for each pictorial expression.
Difficulty Integrating English Language in Mathematical Content (ELL)
• Use a visual in the presentation of math lessons or mathematical ideas. • Provide students with frequent opportunities to draw, use graphs, charts, and flash cards with visual representations. • Use graphic organizers to aid in understanding procedures and key vocabulary. • Make the instructions hands-on to aid in crossing the language barrier. • Put new math terms or vocabulary into context as soon as and as often as possible. • Give students a lot of opportunities to use the new words. • Use direct instruction to demonstrate the new skill first, then have students do it with you, then on their own (I do it; we do it; they do it). • Use guided practice until students perform the task in independent practice. • Encourage group-based work to aid in developing language component of lesson. • Simplify instructions so that students do not get stuck on the directions. • Write clearly in print, rather than cursive. • Incorporate students' names into lesson problems to aid students with concrete examples. • Introduce problems in familiar context, to aid students in making sense of concept or structure of the problem.

Accommodations Support in Mathematics	
Calculation Devices (CD) **Visual Organizers (VO)** **Graphic Organizers (GO)**	**Math Disabilities**
• Calculator **(CD)** • Number line **(CD)** • Multiplication chart, arithmetic table, number chart **(CD)** • Graphic organizers highlighting steps or new math word **(GO)** • Templates for recording information **(VO)**	• Student has difficulty retrieving math facts, or the steps of algorithms in memory. Has difficulty retrieving information, has problems using math vocabulary and making connections among concepts learned in prior months. • Student has difficulty keeping track of calculations in multi-step problems or performing mental calculations.
• Color cubes, color tiles, attribute blocks, numeral cards, number cubes, pattern blocks, tangrams, dominoes, color tiles **(CD)** • Larger or partially filled-in templates **(VO)**	• Student has difficulty with conceptually visualizing and identifying patterns and data analysis with probability. • Student has difficulty processing visual-spatial patterns to determine rules and analyzing graphical representations of functions. • Student has problems working with tables and graphs.
• Compasses, protractors, rulers **(CD)** • Miras, geoboards, tangrams pentominoes, geometric solids polyhedra models **(CD)** • Cubes, color tiles **(CD)** • Graphic organizers **(GO)** • Templates, graph paper and diagrams **(VO)**	• Students have visual-spatial difficulties (e.g., problems involving space in areas, as required in Algebra & Geometry). • Student has problems processing non-verbal information. Therefore, they have problems perceiving spatial relationships, such as drawing and copying geometric forms and designs.
• Algebra tiles **(CD)** • Base-ten materials **(CD)** • Math balance, 10-frames **(CD)** • Dominoes, 2-color counters **(CD)** • Calculator **(CD)** • Graphic organizers of procedures and key points **(GO)**	• Student has problems conceptually processing abstract concepts such as variables and linear functions, equations/inequalities, factoring polynomials, and integer operations.
• Decimal squares **(CD)** • Fraction models **(CD)** • Number cards, base ten blocks **(CD)** • Calculator **(CD)** • Base-ten template **(VO)**	• Student has difficulty with visual-spatial representation, decimals, equivalence computation such as converting and making connections from fractions to decimals to percents.
• Clocks, time line ruler **(CD)** • Number line, hundredths chart or numbers chart **(CD)**	• Student has spatial temporal difficulty (e.g., telling time, recognizing before/after and next to/above and noticing size differences).

Mathematics and Careers

Mathematics and careers must be infused into the discipline as mandated by both the Maryland State Department of Education and the Prince George's County Board of Education. Knowledge of mathematics most assuredly enhances one's career opportunities. In the United States, the use of mathematics in numerous jobs and careers stresses the need for a good working knowledge of math as being imperative in everyday life. The infusion of mathematics in real-world application as it relates to career paths has been addressed throughout this document through the use of quarterly projects and unit lessons.

Suggested Career Education Activities:

(Teachers should keep in mind both paid and volunteer work--vocational and service-oriented.)

1. Teacher might ask students to write hobbies, areas of expertise, interests, and career aspirations on a 5 x 7 card at the beginning of the year. This information should help individualize activities and instruction. Students may be "consultants" to the class.

2. Students and teacher examine lessons in text to see how material can be related to occupational situations.

3. Students might interview workers in the school (teachers, media specialist, counselors, and custodians) inquiring as to how they use knowledge of mathematics.

4. Students prepare occupational packet for use by younger students in career exploration (e.g., a metric packet).

5. Students collect and study "want ads" for a variety of occupations available and identify knowledge of mathematics as being a requirement.

Multicultural Education

Education that is multicultural is education that promotes the recognition, understanding, and acceptance of individual uniqueness, interdependence, and cultural diversity within a pluralistic society. A multicultural education also gives all students opportunities to "see themselves" in the curriculum in positive ways, and on a continuing basis. The term "multicultural" as used here refers broadly to the many cultural groups within our nation and our world: racial, ethnic, regional, religious, and socioeconomic groups, as well as males and females, the young and the old and disabled persons.

Suggested Multicultural Education Activities as they relate to mathematics lessons and/or strategies:

1. Read mathematics literature that highlights different cultures, ethnicities, religions, and socioeconomic groups.
2. Highlight the mathematic achievements of various genders, ages, races, and persons with disabilities.
3. Represent various genders, cultures, ethnicities, ages, and people with disabilities when formatting word problems.
4. Have students to research different cultures and ethnicities and present their findings to the class. Allow students to share (after teacher review) strategies from various cultures for different mathematical algorithms/processes.
5. Introduce foods, music, and art from other cultures in order for students to gain an appreciation and understanding of their diverse world.
6. Teachers may invite guest speakers from other countries to share their values, customs, and beliefs with students.
7. Show how geometric patterns are infused in various cultures.
8. Have students participate in math games in other cultures. Have them compare and contrast the differences in those games with similar games they are familiar with.
9. Show students how different cultures use manipulatives to solve math problems.
10. Show the different currency in other cultures and compare the currencies to the US dollar.

Maryland Technology Literacy Standards for Students

Standard 1.0 – Technology Systems: Develop foundations in the understanding and uses of technology systems

Standard 2.0 – Digital Citizenship: Demonstrate an understanding of the history of technology and its impact on society, and practice ethical, legal, and responsible use of technology to assure safety

Standard 3.0 – Technology for Learning and Collaboration: Use a variety of technologies for learning and collaboration

Standard 4.0 – Technology for Communication and Expression: Use technology to communicate information and express ideas using various media formats

Standard 5.0 – Technology for Information Use and Management: Use technology to locate, evaluate, gather, and organize information and data

With the acceptance of the Maryland Educational Technology Plan for the New Millennium: 2007-2012, by the State Board of Education on April 24, 2007, the State now has technology literacy standards for students, teachers and administrators.

These standards define what students, teachers and administrators need to know and be able to do using technology. Click on the link below to view each set of standards by grade level and objectives.

http://www.marylandpublicschools.org/MSDE/programs/technology/techstds/

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CURRICULUM GUIDE EVALUATION FORM

This form is to be used by individual teachers to provide a reaction to the curriculum guide currently being used. At the end of each unit taught or after teaching from the entire document, please complete the form and send it to the content area supervisor. Your input is necessary in order to assess what revisions must be made in the document. Thank you for helping to review and revise your curriculum so that it is meaningful to your teaching.

_____

Name of Document

_____

Name of Unit/Chapter Evaluated

_____ _____

Instructional/Grade Level Publication Date

1. In-service was received on this publication. Yes _____ No _____

2. The in-service was (adequate, inadequate) for using this document. Yes _____ No _____

3. Teachers could use further in-service on the following topics/chapters/units:

_____

_____

4. The errors/omissions noted in the document are on page(s)_____

_____

5. The best written and most helpful sections or pages of this document are:

_____

_____

6. Information needs to be revised on the following:

_____

_____

7. The attached material (outline, lesson plan, etc.) should be added to the document.

_____

8. Did the format of the guide make it easy to use? Yes _____ No _____

9. What changes would you like to see included? _____

_____

10. Do the lessons contain realistic teaching time frames? Yes _____ No _____

11. Are there a sufficient number of teaching lessons/activities? Yes ____ No ____

12. Are there a sufficient number of available resources listed? Yes ____ No ____

13. Was the content appropriate for the level of teaching? Yes ____ No _____

14. Does the content adequately provide for Title IX (sex equality) guidelines? Yes ____ No ____

15. Does the content adequately provide for inclusion of information about multi-cultural and multiracial relationships?

Yes ____ No ____

16. The following suggestions would improve this document: _____

_____

_____

_____

_____

Name (if desired) _____

School _____